This book presents original mathematical models of thermal stresses in composite materials with three components. In contrast to mathematical models for two-component materials, which are determined in the first volume, the three-component materials consist of an isotropic matrix and isotropic ellipsoidal inclusions with an isotropic ellipsoidal envelope. These stresses are a consequence of different thermal expansion coefficients of the material components. The mathematical determination results from mechanics of an isotropic elastic continuum, and results in different mathematical solutions. Due to these different mathematical solutions, the principle of minimum elastic energy is considered. The mathematical models for the three-component materials are applicable to those for the thermal-stress induced micro-/macro-strengthening and crack formation in the two-component materials, which are determined in the first volume.

Dr. Ladislav Ceniga, DSc. (Institute of Materials Research, Slovak Academy of Sciences, Kosice, Slovak Republic) works on mathematical models of stresses in composites.

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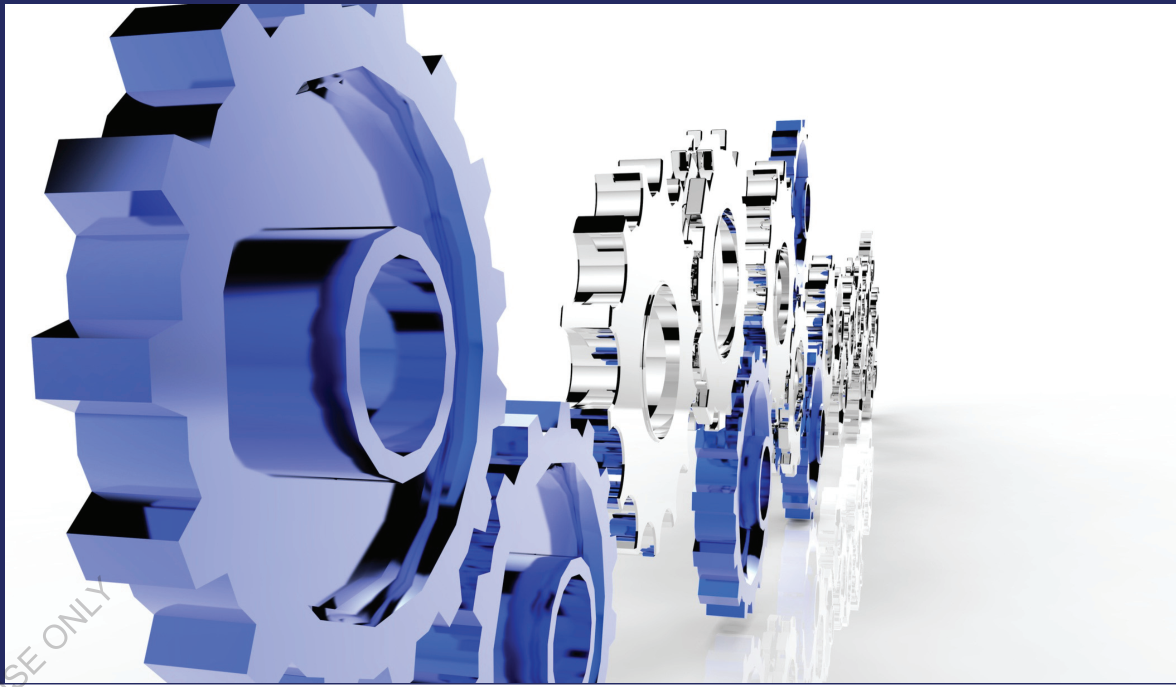
Thermal Stress in Composite Ellipsoids

Ladislav Ceniga



Ladislav Ceniga

Thermal Stresses in Composites with Ellipsoidal Inclusions II

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Thermal Stresses in Composites with Ellipsoidal Components II

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Táto kniha je venovaná s láskou mojim najdrahším rodičom a starým rodičom.

This book is dedicated with love to my dearest parents and grandparents.

Contents

Introduction

This book[1] presents original mathematical models of thermal stresses in composite materials with three components. In contrast to mathematical models of the thermal stresses in two-component materials, which are determined in the first volume [1], the three-component materials consist of an isotropic matrix and isotropic ellipsoidal inclusions with an isotropic ellipsoidal envelope. These stresses originate during a cooling process, and are a consequence of different thermal expansion coefficients of the matrix, the ellipsoidal envelope and the ellipsoidal inclusion.

The mathematical models are determined for a suitable model system. The model system is required to correspond to real isotropic matrix-envelope-inclusion composites. The thermal stresses are derived within a suitable coordinate system. The coordinate system is required to correspond to a shape of the ellipsoidal envelope and inclusion. Consequently, the mathematical determination results from mechanics of an isotropic elastic continuum, and results in different mathematical solutions for the thermal stresses in the material components. Due to these different mathematical solutions, the principle of minimum elastic energy is considered.

The mathematical models for the three-component materials are applicable to those for the thermal-stress induced micro-/macro-strengthening and crack formation in the two-component materials, which are determined in the first volume [1].

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Chapter 1

Matrix-Envelope-Inclusion Composite

1.1 Model System

Figure 1.1 shows a model system, corresponding to real matrix-envelope-inclusion composites, which is considered within the mathematical models of the thermal stresses. This model system consists of an infinite isotropic matrix with isotropic ellipsoidal envelopes on a surface of isotropic ellipsoidal inclusions with the dimensions a_{1q}, a_{2q}, a_{3q} ($q = IN,E$) and the inter-inclusion distance d along the axes x_1, x_2, x_3 of the Cartesian system $(Ox_1x_2x_3)$, respectively, where O represents a centre of the ellipsoidal inclusion. The subscript $q = IN$ and $q = E$ is related to the inclusion and envelope, respectively.

As presented in [2]–[23], the thermal stresses are determined in the cubic cells with the dimension d along the axes x_1, x_2, x_3 and with central ellipsoidal inclusions (see Figure 1.2). Due to the infinite matrix, the thermal stresses, which are determined for one of the cubic cells, are identical with those, which are determined for any of the cubic cells [2]–[23]. With regard to the volume $V_{IN} = 4\pi a_1, a_2 a_3$ [24] and $V_C = d^3$ of the ellipsoidal inclusion and the cubic cell, the inter-inclusion distance d as a function of the inclusion volume fraction v_{IN} is derived as

$$v_{IN} = \frac{V_{IN}}{V_C} = \frac{4\pi a_{1IN} a_{2IN} a_{3IN}}{3d^3} \in \left(0, \frac{\pi}{6}\right), \quad d = \left(\frac{4\pi a_{1IN} a_{2IN} a_{3IN}}{3v_{IN}}\right)^{1/3}, \qquad (1.1)$$

where the value $v_{INmax} = \pi/6$ results from the condition $a_{iIN} \to a_{iE} \to d/2$ ($i = 1,2,3$). Accordingly, the thermal stresses are functions of the material parameters a_{iq} ($i = 1,2,3$ $q = IN,E$), v_{IN}, d.

1.2 Coordinate System

Figure 1.3 shows the ellipse E with the dimensiions a, b along the axes x, y, respectively. The ellipse E is described by the function

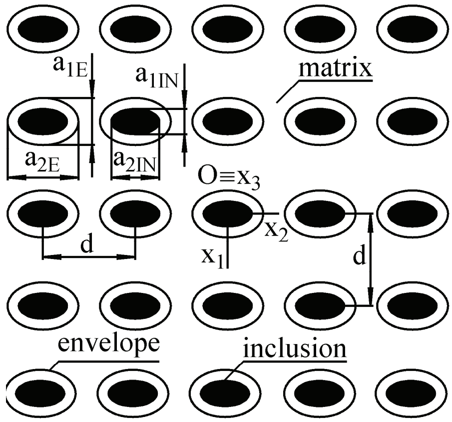


Figure 1.1: The matrix-envelope-inclusion system with an infinite isotropic matrix with isotropic ellipsoidal envelopes on a surface of isotropic ellipsoidal inclusions with the dimensions a_{1q}, a_{2q}, a_{3q} ($q = IN,E$) and the inter-inclusion distance d along the axes x_1, x_2, x_3 of the Cartesian system $(Ox_1x_2x_3)$, respectively, where O represents a centre of the ellipsoidal inclusion. The subscript $q = IN$ and $q = E$ is related to the inclusion and envelope, respectively.

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1. \tag{1.2}$$

Any point P of the ellipse E is described by the coordinates [24]

$$x = a\sin\nu, \quad y = b\cos\nu, \quad \nu \in \langle 0, 2\pi \rangle, \tag{1.3}$$

where the normal n of the ellipse E at the point P is derived [24]

$$\frac{\partial x}{\partial \nu}(x - a\sin\nu) + \frac{\partial y}{\partial \nu}(y - b\cos\nu) = 0. \tag{1.4}$$

With regard to Equations (1.3), (1.4), we get

$$yb\sin\nu - xa\cos\nu + \left(a^2 - b^2\right)\sin\nu\cos\nu = 0. \tag{1.5}$$

The thermal stresses are determined by the spherical coordinates $(r, \varphi\nu)$ (see Figure 1.4). The model system in Figures (1.1), (1.2) is symmetric, and then the thermal stresses are determined within the intervals $\varphi \in \langle 0, \pi/2 \rangle$, $\nu \in \langle 0, \pi/2 \rangle$ [2]–[23].

Figure 1.4 shows the ellipsoidal inclusion for $\varphi, \nu \in \langle 0, \pi/2 \rangle$ with the centre O and with the dimensions $a_{1IN} = O1$, $a_{2IN} = O2$, $a_{3IN} = O3$ along the axes x_1, x_2,

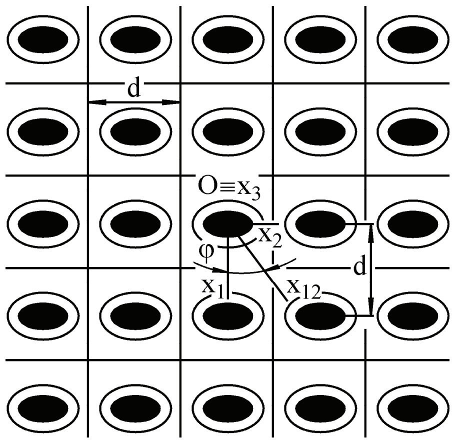


Figure 1.2: The cubic cells with the dimension d along the axes x_1, x_2, x_3 of the Cartesian system $(Ox_1x_2x_3)$ and with the plane $x_{12}x_3$, where O represents a centre of the ellipsoidal inclusion, and ($x_{12} \subset x_1x_2$, $x_{12}x_3 \perp x_1x_2$.

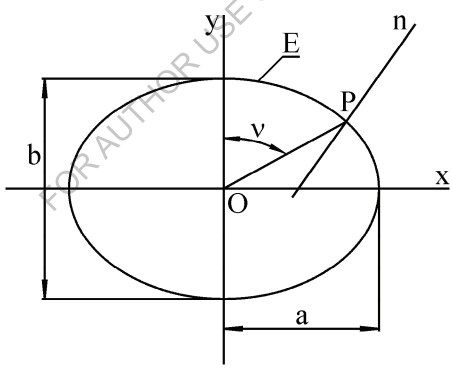


Figure 1.3: The ellipse E with the dimensions a, b along the axes x, y of the Cartesian system (Oxy), respectively, and the point P related to the angle α.

x_3 of the Cartesian system (O, x_1, x_2, x_3) (see Figures (1.1), (1.2)), respectively. Due to clarity of Figure 1.4 the ellipsoidal envelope in Figures 1.1, 1.2 is not depicted in Figure 1.4. Finally, $(P_{IN}, x_n, x_\varphi, x_\nu)$ is a Cartesian system at the point P_{IN}, where the axes x_n and x_ν represents a normal and a tangent of the ellipse E_{IN} at the point P_{IN}, respectively, $x_{12}x_3 \perp x_1x_2$, ($x_{12} \subset x_1x_2$, $x_\varphi \perp x_{12}$.

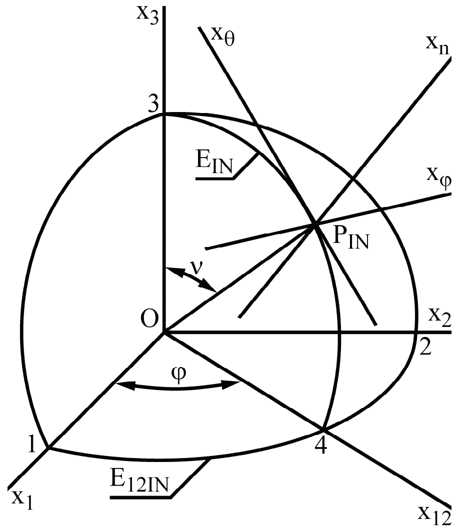
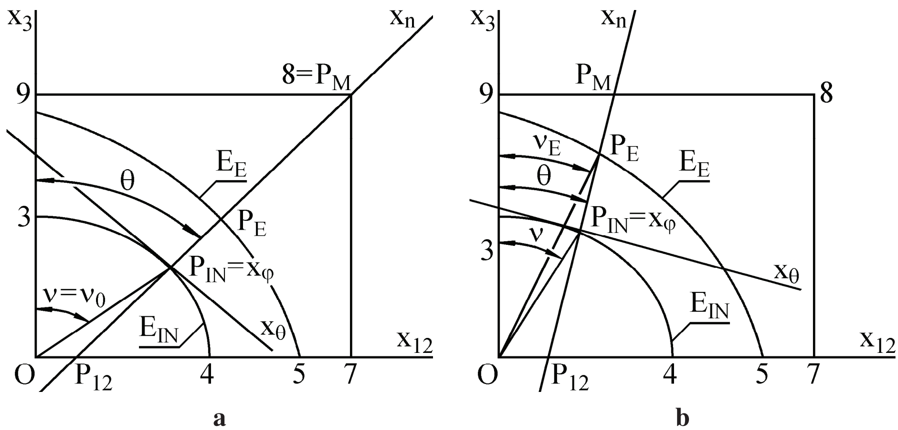


Figure 1.4: The inclusion with the centre O and with the dimensions $a_{1IN} = O1$, $a_{2IN} = O2$, $a_{3IN} = O3$ along the axes x_1, x_2, x_3 of the Cartesian system (O, x_1, x_2, x_3), respectively. The ellipses E_{12}, E_{123} in the planes x_1x_2, $x_{12}x_3$ (see Figure 1.4) are given by Equations (1.6), (1.7), respectively, where $x_{12}x_3 \perp x_1x_2$, ($x_{12} \subset x_1x_2$, $x_\varphi \perp x_{12}$. The point P_{IN} on the inclusion surface is defined by $\varphi, \nu \in \langle 0, \pi/2 \rangle$, $\nu \in \langle 0, \pi/2 \rangle$, and $\left(P_{IN}, x_n, x_\varphi, x_\nu\right)$ is a Cartesian system at the point P_{IN}, where $P_{IN} \subset E_{IN}$. The axes x_n and x_ν represents a normal and a tangent of the ellipse E_{IN} at the point P_{IN}, respectively. Due to clarity of Figure 1.4 the ellipsoidal envelope in Figures 1.1, 1.2 is not depicted in Figure 1.4.

Figure 1.5 shows the cross section $O789$ of the cubic cell in the plane $x_{12}x_3$ (see Figures 1.2, 1.4). The angle $\nu \in \langle 0, \pi/2 \rangle$ defines a position of the point P_{IN} with the Cartesian system $\left(P_{IN}, x_n, x_\varphi, x_\nu\right)$ (see Figure 1.4) for $\nu = \nu_0$ (see Figure 1.5a), $\nu \in \langle 0, \nu_0)$ (see Figure 1.5b), $\nu \in (\nu_0, \pi/2\rangle$ (see Figure 1.5c). The points P_{12}, P_M represent intersections of the normal x_n with $O789$. Due to clarity of Figure 1.5, the angle $\nu_E = \angle(OP_E, x_3)$ (see Equation (1.14)) is depicted in Figure 1.5b only.

Let x_{nIN} and x_{nE} represent normals on the envelope-inclusion and envelope-matrix boundaries at the points P_{IN} and P_E, respectively. As analysed in [2]-[21], due to the symmetry of the model system, any points P_{IN} and P_E on the envelope-inclusion and envelope-matrix boundaries (see Figure 1.5) exhibit the displacements u_{nIN} and u_{nE} along the normals x_{nIN} and x_{nE}, respectively, i.e., $u_{\varphi q} = u_{\nu q} = 0$ ($q = IN, E$) [2]-[21], where $u_{\varphi q}$, $u_{\nu q}$ are displacements along the tangents $x_{\varphi q}$, $x_{\nu q}$, respectively.



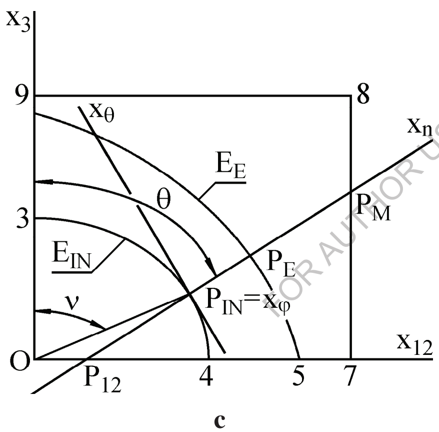


Figure 1.5: The angle $\nu \in \langle 0, \pi/2 \rangle$ defines a position of the point P_{IN} with the Cartesian system $(P_{IN}, x_n, x_\varphi, x_\nu)$ (see Figure 1.4) for (a) $\nu = \nu_0$, (b) $\nu \in \langle 0, \nu_0)$, (c) $\nu \in (\nu_0, \pi/2\rangle$, where ν_0 is given by Equation (1.9). The points P_{12}, P_M represent intersections of the normal x_n with $O789$, where $O789$ is a cross section of the cubic cell in the plane $x_{12}x_3$ (see Figures 1.2, 1.4). The angle $\theta = \angle(x_n, x_3)$ is given by Equation (1.18). Due to clarity of Figure 1.5, the angle $\nu_E = \angle(OP_E, x_3)$ (see Equation (1.14)) is depicted in Figure 1.5b only.

Consequently, the mathematical models in this book can be determined provided that $u_{\varphi q} = u_{\nu q} = 0$ ($q = IN, E$), which results in the condition $x_n = x_{nIN} = x_{nE}$. Finally, the condition $x_n = x_{nIN} = x_{nE}$ defines relationships between the dimensions a_{iIN} and a_{iE} ($i = 1,2,3$; see Equation (1.13)). With regard to Equation (1.13), the ellipsoidal inclusion and ellipsoidal envelope are required to represent a rotational ellipsoid (a spheroid), i.e., $a_{1q} = a_{2q} \neq a_{3q}$ ($q = IN, E$). The relationships (see Equation (1.13)) are determined by the following procedure.

With regard to Equation (1.3), any point of the ellipse E_{12IN} in the plane x_1x_2 is described by the coordinates

$$x_1 = a_{1IN}\cos\varphi, \quad x_2 = a_{2IN}\sin\varphi, \quad \varphi \in \left\langle 0, \frac{\pi}{2} \right\rangle. \tag{1.6}$$

Similarly, any point P_{IN} of the ellipse E_{IN} in the plane $x_{12}x_3$ (see Figure 1.5) is described by the coordinates

$$\begin{aligned} &x_{12IN} = a_{12IN}\sin\nu, \quad x_{3IN} = a_{3IN}\cos\nu, \\ &a_{12IN} = O4 = \sqrt{a_{1IN}^2\cos^2\varphi + a_{2IN}^2\sin^2\varphi}, \quad \varphi, \nu \in \left\langle 0, \frac{\pi}{2} \right\rangle. \end{aligned} \tag{1.7}$$

Consequently, any point P_E of the ellipse E_E in the plane $x_{12}x_3$ (see Figure 1.5) is described by the coordinates

$$\begin{aligned} &x_{12E} = a_{12E}\sin\nu_E, \quad x_{3E} = a_{3E}\cos\nu_E, \\ &a_{12E} = O5 = \sqrt{a_{1E}^2\cos^2\varphi + a_{2E}^2\sin^2\varphi}, \quad \varphi, \nu_E \in \left\langle 0, \frac{\pi}{2} \right\rangle. \end{aligned} \tag{1.8}$$

With regard to Equation (1.5), (1.7), (1.8), the normals x_{nIN} and x_{nE} at the points P_{IN} and P_E of the ellipses E_{IN} and E_E in the plane $x_{12}x_3$ (see Figure 1.5), respectively, are derived as

$$x_3\, a_{3IN}\,\sin\nu - x_{12}\, a_{12IN}\,\cos\nu + \left(a_{12IN}^2 - a_{3IN}^2\right)\sin\nu\,\cos\nu = 0, \quad \nu \in \left\langle 0, \frac{\pi}{2} \right\rangle, \tag{1.9}$$

$$x_3\, a_{3E}\,\sin\nu_E - x_{12}\, a_{12E}\,\cos\nu_E + \left(a_{12E}^2 - a_{3E}^2\right)\sin\nu_E\,\cos\nu_E = 0, \quad \nu_E \in \left\langle 0, \frac{\pi}{2} \right\rangle. \tag{1.10}$$

With regard to Equations (1.9), (1.10), the condition $x_{nIN} = x_{nE}$ results in

$$\frac{a_{12IN}^2 - a_{3IN}^2}{a_{12IN}\, a_{3IN}} = \frac{a_{12E}^2 - a_{3E}^2}{a_{12E}\, a_{3E}}, \tag{1.11}$$

$$\tan\nu_E = \frac{a_{12E}\, a_{3IN}}{a_{12IN}\, a_{3E}}\tan\nu. \tag{1.12}$$

With regard to a_{12q} (see Equations (1.7), (1.8)) and due to $a_{3q} \neq f(\varphi)$ $(q = IN,E)$, the relationships between the dimensions a_{iIN} and a_{iE} $(i = 1,2,3)$ are derived as

$$\frac{a_{1IN}^2 - a_{3IN}^2}{a_{1IN}\, a_{3IN}} = \frac{a_{1E}^2 - a_{3E}^2}{a_{1E}\, a_{3E}}, \quad a_{1q} = a_{2q} \neq a_{3q}, \ \ a_{iIN} < a_{iE}, \ \ i = 1,2,3, \ \ q = IN,E, \tag{1.13}$$

where the angle $\nu_E = \angle(OP_E, x_3)$ (see Figure 1.5b, Equation (1.12)) has the form

$$\sin\nu_E = \frac{a_{1E}\,a_{3IN}\,\tan\nu}{\sqrt{(a_{1IN}\,a_{3E})^2 + (a_{1E}\,a_{3IN}\,\tan\nu)^2}},$$
$$\cos\nu_E = \frac{a_{1IN}\,a_{3E}}{\sqrt{(a_{1IN}\,a_{3E})^2 + (a_{1E}\,a_{3IN}\,\tan\nu)^2}}. \tag{1.14}$$

With regard to Equation (1.9), the coordinates $x_{x_{12,0}}$, x_3 of the point P_{12} have the forms

$$x_{120} = \frac{(a_{1IN}^2 - a_{3IN}^2)\sin\nu}{a_{1IN}}, \quad x_3 = 0, \quad \nu \in \left\langle 0, \frac{\pi}{2} \right\rangle. \tag{1.15}$$

Similarly, the coordinates $x_{x_{12M}}$, x_{3M} of the point P_M in Figure 1.5b for $\nu \in \langle 0, \nu_0 \rangle$ are derived as

$$x_{12M} = \frac{\sin\nu}{a_{1IN}} \left(\frac{d\cos\nu}{2\,a_{3IN}} + a_{1IN}^2 - a_{3IN}^2 \right), \quad x_{3M} = \frac{d}{2}, \quad \nu \in \langle 0, \nu_0 \rangle. \tag{1.16}$$

The coordinates $x_{x_{12M}}$, x_{3M} of the point P_M in Figure 1.5c for $\nu \in \langle \nu_0, \pi/2 \rangle$ have the forms

$$x_{12M} = \frac{d}{2\,f(\varphi)\sin\nu},$$
$$f(\varphi) = \cos\varphi, \quad \varphi \in \left\langle 0, \frac{\pi}{4} \right\rangle; \quad f(\varphi) = \sin\varphi, \quad \varphi \in \left\langle \frac{\pi}{4}, \frac{\pi}{2} \right\rangle,$$
$$x_{3M} = \frac{\cos\nu}{a_{3IN}} \left[\frac{a_{1IN}\,d}{2\,f(\varphi)\sin\nu} + a_{3IN}^2 - a_{1IN}^2 \right], \quad \nu \in \left\langle \nu_0, \frac{\pi}{2} \right\rangle. \tag{1.17}$$

The coordinate x_{12M} of the point P_M in Figure 1.5a for $\nu = \nu_0$ is given by Equation (1.17), where $x_{3M} = d/2$. With regard to Equation (1.9), the angle ν_0 represents a root of the following equation

$$\frac{\cos\nu_0}{a_{3IN}} \left[\frac{a_{1IN}\,d}{2\,f(\varphi)\sin\nu_0} + a_{3IN}^2 - a_{1IN}^2 \right] - \frac{d}{2} = 0,$$
$$f(\varphi) = \cos\varphi, \quad \varphi \in \left\langle 0, \frac{\pi}{4} \right\rangle; \quad f(\varphi) = \sin\varphi, \quad \varphi \in \left\langle \frac{\pi}{4}, \frac{\pi}{2} \right\rangle, \tag{1.18}$$

and this root is determined by a numerical method. The angle $\theta = \angle(x_n, x_3)$ is derived as

$$\cos\theta = \frac{x_{3P}}{\sqrt{(x_{12P} - x_{121})^2 + x_{3P}^2}} = \frac{1}{\sqrt{1 + (a_{3IN}\tan\nu/a_{1IN})^2}},$$
$$\sin\theta = \frac{1}{\sqrt{1 + (a_{1IN}\cot\nu/a_{3IN})^2}}. \tag{1.19}$$

Consequently, we get [24]

$$\frac{\partial}{\partial\theta} = \left(\frac{\partial\theta}{\partial\varphi}\right)^{-1}\frac{\partial}{\partial\varphi} = \Theta\frac{\partial}{\partial\varphi}, \tag{1.20}$$

where the function $\Theta = \Theta(\varphi)$ has the form

$$\Theta = \left(\frac{a_{1IN}}{a_{3IN}}\right)\left[\left(\frac{a_{3IN}\sin\nu}{a_{1IN}}\right)^2 + \cos^2\nu\right]. \tag{1.21}$$

As presented in [2]–[23], the thermal stresses, which are determined along the axes x_n, x_φ, x_θ of the Cartesian system $(P, x_n, x_\varphi, x_\theta)$, represent function of the spherical coordinates (x_n, φ, θ) for $\varphi, \theta \in \langle 0, \pi/2\rangle$. The intervals $x_n \in \langle 0, x_{IN}\rangle$, $x_n \in \langle x_{IN}, x_E\rangle$ and $x_n \in \langle x_E, x_M\rangle$ are related to the ellipsoidal inclusion, envelope and cell matrix, respectively. Finally, we get

$$x_{IN} = P_{12}P_{IN} = \sqrt{(a_{1IN}\sin\nu - x_{120})^2 + (a_{3IN}\cos\nu)^2},$$
$$x_E = P_{IN}P_E = \sqrt{(a_{1E}\sin\nu_E - a_{1IN}\sin\nu)^2 + (a_{3E}\cos\nu_E - a_{3IN}\cos\nu)^2},$$
$$x_M = P_E P_M = \sqrt{(x_{12M} - a_{1E}\sin\nu_E)^2 + (x_{3M} - a_{3E}\cos\nu_E)^2}, \tag{1.22}$$

where $\sin\nu_E$, $\cos\nu_E$ and x_{120}, x_{12M}, x_{3M} are given by Equations (1.12) and (1.15)–(1.16), respectively.

Chapter 2

Elastic Solid Continuum

2.1 Fundamental Equations

As analysed in [2]-[21], any point P of the normal x_n exhibits the displacement u_n along x_n. The thermal stresses are determined along the axes x_n, x_φ, x_θ of the Cartesian system $(P, x_n, x_\varphi, x_\theta)$. Fundamental equations of mechanics of a solid continuum are represented by Cauchy's equations, the equilibrium equations and Hooke's law. Cauchy's equations represent functions of strains and displacements. With respect to the normal displacement u_n, Cauchy's equations have the forms [2]-[21, 23]

$$\varepsilon_n = \frac{\partial u_n}{\partial x_n}, \tag{2.1}$$

$$\varepsilon_\varphi = \varepsilon_\theta = \frac{u_n}{x_n}, \tag{2.2}$$

$$\varepsilon_{n\varphi} = \varepsilon_{\varphi n} = \frac{1}{x_n} \frac{\partial u_n}{\partial \varphi}, \tag{2.3}$$

$$\varepsilon_{n\theta} = \varepsilon_{\theta n} = \frac{\Theta}{x_n} \frac{\partial u_n}{\partial \nu}, \tag{2.4}$$

where ε_n is a normal strain along the axis x_n, and Θ is given by Equation (1.21). Consequently, ε_φ and ε_θ are tangential strains along the axes x_φ and x_θ, respectively. Finally, $\varepsilon_{n\varphi}$, $\varepsilon_{n\theta}$ and $\varepsilon_{\varphi n}$, $\varepsilon_{\theta n}$ represent shear strains along the axes x_n and x_φ, x_θ, respectively. Due to $u_\varphi = u_\nu = 0$, we get $\varepsilon_{\varphi\nu} = \varepsilon_{\nu\varphi} = 0$ [2]–[23], where u_φ, u_ν are displacements along the axes x_φ, x_ν, respectively, and $\varepsilon_{\varphi\nu}$ is a shear strain. As presented in [2]–[23], the equilibrium equations are derived as

$$2\sigma_n - \sigma_\varphi - \sigma_\nu + x_n \frac{\partial \sigma_n}{\partial x_n} + \frac{\partial \sigma_{n\varphi}}{\partial \varphi} + \Theta \frac{\partial \sigma_{n\theta}}{\partial \nu} = 0, \tag{2.5}$$

$$\frac{\partial \sigma_\varphi}{\partial \varphi} + 3\sigma_{n\varphi} + x_n \frac{\partial \sigma_{n\varphi}}{\partial x_n} = 0, \tag{2.6}$$

$$\Theta \frac{\partial \sigma_\theta}{\partial \nu} + 3\sigma_{n\theta} + x_n \frac{\partial \sigma_{n\theta}}{\partial x_n} = 0, \tag{2.7}$$

where σ_n is a normal stess along the axis x_n. Consequently, σ_φ and σ_θ are tangential stresses along the axes x_φ and x_θ, respectively. Finally, $\sigma_{n\varphi}$, $\sigma_{n\theta}$ and $\sigma_{\varphi n}$, $\sigma_{\theta n}$ represent shear stresses along the axes x_n and x_φ, x_θ, respectively, where $\sigma_{n\varphi} = \sigma_{\varphi n}$, $\sigma_{n\theta} = \sigma_{\theta n}$. Due to $\varepsilon_{\varphi\nu} = \varepsilon_{\nu\varphi} = 0$, we get $\sigma_{\varphi\nu} = \sigma_{\nu\varphi} = 0$ [2]–[23], where $\sigma_{\varphi\nu}$ is a shear stress. With regard to $\varepsilon_{\varphi\theta} = 0$, $\sigma_{\varphi\theta} = 0$, Hooke's law has the form [2]–[21, 23]

$$\varepsilon_n = s_{11}\sigma_n + s_{12}\left(\sigma_\varphi + \sigma_\theta\right), \tag{2.8}$$

$$\varepsilon_\varphi = s_{12}\left(\sigma_n + \sigma_\theta\right) + s_{11}\sigma_\varphi, \tag{2.9}$$

$$\varepsilon_\theta = s_{12}\left(\sigma_n + \sigma_\varphi\right) + s_{11}\sigma_\theta, \tag{2.10}$$

$$\varepsilon_{n\theta} = s_{44}\sigma_{n\theta}, \tag{2.11}$$

$$\varepsilon_{n\varphi} = s_{44}\sigma_{n\varphi}, \tag{2.12}$$

where s_{11}, s_{12}, s_{44} are derived as [25]

$$s_{11} = \frac{1}{E}, \quad s_{12} = -\frac{\mu}{E}, \quad s_{44} = \frac{2\left(1+\mu\right)}{E}. \tag{2.13}$$

Finally, E and μ are Young's modulus and Poisson's ratio, respectively. In case of the ellipsoidal inclusion and the cell matrix, we get $E = E_{IN}$, $\mu = \mu_{IN}$ and $E = E_M$, $\mu = \mu_M$, respectively. With regard to Equations (2.1)–(2.4), (2.8)–(2.12), we get [2]–[23]

$$\sigma_n = \left(c_1 + c_2\right)\frac{\partial u_n}{\partial x_n} - 2c_2\frac{u_n}{x_n}, \tag{2.14}$$

$$\sigma_\varphi = \sigma_\theta = -c_2\frac{\partial u_n}{\partial x_n} + c_1\frac{u_n}{x_n}, \tag{2.15}$$

$$\sigma_{n\varphi} = \frac{1}{s_{44}x_n}\frac{\partial u_n}{\partial \varphi}, \tag{2.16}$$

$$\sigma_{n\theta} = \frac{\Theta}{s_{44}x_n}\frac{\partial u_n}{\partial \nu}, \tag{2.17}$$

where c_1, c_2, c_3 (see Equation (2.20)) have the forms

$$c_1 = \frac{E}{\left(1+\mu\right)\left(1-2\mu\right)}, \quad c_2 = -\frac{\mu E}{\left(1+\mu\right)\left(1-2\mu\right)}, \quad c_3 = -4\left(1-\mu\right) < 0, \tag{2.18}$$

and $c_3 < 0$ due to $\mu < 0.5$ for real isotropic components [26]. Let Equations (2.14)–(2.17) be substituted to Equation (2.18) and to $[\partial \mathrm{Eq.}(2.6)/\partial\varphi] + \Theta\,[\partial \mathrm{Eq.}(2.7)/\partial\nu]$. Consequently, Equations (2.5)–(2.7) are derived as

$$x_n^2\,\frac{\partial^2 u_n}{\partial x_n^2} + 2\,x_n\,\frac{\partial u_n}{\partial x_n} - 2\,u_n + \frac{U_n}{s_{44}\,(c_1 + c_2)} = 0, \tag{2.19}$$

$$x_n\,\frac{\partial U_n}{\partial x_n} = c_3\,U_n, \tag{2.20}$$

where U_n is derived as

$$U_n = \frac{\partial^2 u_n}{\partial\varphi^2} + \Theta^2\,\frac{\partial^2 u_n}{\partial\nu^2}. \tag{2.21}$$

The system of the differential equations (2.19), (2.21) is solved by the mathematical procedures in Sections 3.1, 4.1, 5.1, 6.1, 7.1.

2.2 Elastic Energy

As analysed in [2]–[23] with respect to the different mathematical procedures (see Sections 3.1, 4.1, 5.1, 6.1, 7.1), such a mathematical solution, which exhibits a minimum value of the elastic energy W_C of the cubic cell, is considered, where W_{IN}, W_E and W_M is elastic energy, which is accumulated in the volume V_{IN}, V_E and V_M of the ellipsoidal inclusion, envelope and cell matrix, respectively. The elastic energy density w is derived as [25]

$$w = \frac{1}{2}\left(\varepsilon_n\sigma_n + \varepsilon_\varphi\sigma_\varphi + \varepsilon_\theta\sigma_\theta\right) + \varepsilon_{n\varphi}\sigma_{n\varphi} + \varepsilon_{n\theta}\sigma_{n\theta}, \tag{2.22}$$

and W_{IN}, W_E, W_M and W_C have the forms

$$W_{IN} = \int\limits_{V_{IN}} w_{IN}\,dV_{IN} = 8\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\int\limits_0^{x_{IN}} w_{IN}\,x_n^2\,dx_n\,d\varphi\,d\nu,$$

$$W_M = \int\limits_{V_M} w_M\,dV_M = 8\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\int\limits_{x_{IN}}^{x_M} w_M\,x_n^2\,dx_n\,d\varphi\,d\nu,$$

$$W_E = \int\limits_{V_E} w_E\,dV_E = 8\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\int\limits_{x_E}^{x_M} w_E\,x_n^2\,dx_n\,d\varphi\,d\nu,$$

$$W_C = W_{IN} + W_E + W_M. \tag{2.23}$$

2.3 Reason of Thermal Stresses

The thermal stresses are a consequence of the condition $\alpha_{IN} \neq \alpha_E \neq \alpha_M$, $\alpha_{IN} = \alpha_E \neq \alpha_M$, $\alpha_{IN} \neq \alpha_E = \alpha_M$, where α_q is a thermal expansion coefficient of the ellipsoidal inclusion ($q = IN$), the ellipsoidal envelope ($q = E$) and the matrix ($q = M$) (see Figure 1.1). The thermal stresses originate at the temperature $T \leq T_r$, where T_r is relaxation temperature. If $T > T_r$, then the thermal stresses are relaxed by thermal-activated processes [26]. The temperature T_r is defined by the formula $T_r = (0.35 - 0.4) \times T_m$ [26], where T_m is melting temperature of a three-component material. If $\alpha_q = \alpha_q(T)$ ($q = IN,E,M$), then the coefficient β_q is derived as

$$\beta_q = \int_T^{T_r} \alpha_q \, dT, \quad q = IN, E, M. \tag{2.24}$$

The thermal stresses are a consequence of the normal stresses p_{1n}, p_{2n}, which act on the envelope-inclusion and envelope-matrix boundaries for $x_n = x_{IN}$ and $x = x_E$, respectively.

If $\beta_{IN} \neq \beta_E \neq \beta_M$, then p_{1n}, p_{2n} are derived as [2]–[23]

$$p_{1n} = \frac{(\rho_M + \rho_{22E})(\beta_E - \beta_{IN}) + \rho_{21E}(\beta_E - \beta_M)}{(\rho_{IN} + \rho_{11E})(\rho_M + \rho_{22E}) - \rho_{12E}\rho_{21E}}, \tag{2.25}$$

$$p_{2n} = \frac{(\rho_{IN} + \rho_{11E})(\beta_M - \beta_E) + \rho_{12E}(\beta_{IN} - \beta_E)}{(\rho_{IN} + \rho_{11E})(\rho_M + \rho_{22E}) - \rho_{12E}\rho_{21E}}, \tag{2.26}$$

where the coefficients ρ_{IN}; ρ_{ijE} ($i, j = 1,2$); ρ_M are given by Equations (6.89); (6.62), (6.72), (6.82), (7.83), (7.93), (7.103); (3.24), (4.24), (4.34), (4.44), (4.54), (5.23), (5.33), (5.43), (5.53), (6.22), (6.32), (6.42), (6.52), (7.23), (7.33), (7.43), (7.53), (7.63), (7.73), respectively, with respect to minimum W_c (see Equation (2.23)).

If $\beta_{IN} \neq \beta_E = \beta_M$, then the stress p_{1n} has the form [2]–[23]

$$p_{1n} = \frac{\beta_E - \beta_{IN}}{\rho_{IN} + \rho_{1ME}}, \tag{2.27}$$

where ρ_{IN} and ρ_{1ME} are given by Equations (6.89) and (3.53), (4.130), (6.136), respectively, with respect to minimum W_c (see Equation (2.23)).

If $\beta_{IN} = \beta_E \neq \beta_M$, then the stress p_{2n} is derived as [2]–[23]

$$p_{2n} = \frac{\beta_M - \beta_E}{\rho_M + \rho_{2INE}}, \tag{2.28}$$

where ρ_{IN} and ρ_{2INE} are given by Equations (6.89) and (3.63), (4.140), (5.129), (5.139), (6.143), respectively, with respect to minimum W_c (see Equation (2.23)).

As an example of minimum W_c with respect to different mathematical solutions of the thermal stresses in the matrix, the ellipsoidal envelope and inclusion, let a real three-component material with the thermal expansion coefficients $\alpha_{IN} \neq \alpha_E \neq \alpha_M$ be considered. Let W_M, W_E, W_{IN} given by Equations (3.22), (4.72), (6.88), respectively, result in minimum W_c for this real material. Consequently, the thermal stresses in the matrix; the ellipsoidal envelope and inclusion, which are functions of p_{1n}, p_{2n} (see Equations (2.25), (2.26)), are given by Equations (3.15)–(3.24); (4.65)–(4.74) and (6.83)–(6.89), respectively. Finally, the coefficients ρ_M, ρ_{ijE} $(i, j = 1,2)$ and ρ_{IN} in Equations (2.25), (2.26) are given by Equations (3.24), (4.74) and (6.89), respectively.

2.4 Boundary Conditions

The mathematical solutions of the system of the differential equations (2.19), (2.21) include integration constants. As presented in [2]–[23], these constants are determined by the following boundary conditions for the ellipsoidal inclusion, envelope and cell matrix.

2.4.1 Matrix

The absolute value $|u_{nM}|$ is required to represent a decreasing function of $x_n \in \langle x_E, x_M \rangle$ with a maximum on the envelope-matrix boundary, i.e., for $x_n = x_E$.

If $\beta_{IN} = \beta_E \neq \beta_M$ or $\beta_{IN} \neq \beta_E \neq \beta_M$, then the mandatory boundary conditions have the forms [2]–[23]

$$(\sigma_{nM})_{x_n = x_E} = -p_{2n}, \tag{2.29}$$

$$(u_{nM})_{x_n = x_M} = 0, \tag{2.30}$$

where x_E and p_{2n} are given by Equations (1.22) and (2.26), (2.28), respectively. Equations (2.29) and (2.30) represent stress and geometric boundary conditions, respectively.

If $\beta_{IN} \neq \beta_E = \beta_M$, then we get $(u_{nM})_{x_n = x_E} = (u_{nE})_{x_n = x_E}$. With regard to Equation (2.2), we get $(u_{nM})_{x_n = x_E} = x_E \left(\varepsilon_{\varphi E}\right)_{x_n = x_E}$, $(u_{nE})_{x_n = x_E} = x_E \left(\varepsilon_{\varphi E}\right)_{x_n = x_E}$. If $\beta_{IN} \neq$

$\beta_E = \beta_M$, then Equation 2.29 is replaced by the following mandatory boundary condition [2]–[23]

$$\left(\varepsilon_{\varphi M}\right)_{x_n = x_E} = \left(\varepsilon_{\varphi E}\right)_{x_n = x_E} = -p_{1n}\rho_{2ME}, \tag{2.31}$$

where ρ_{2ME} is given by Equations (3.53), (4.130), (6.135) with respect to minimum W_c (see Equation (2.23)). In this case, two integration constants for the cell matrix, which are included in $\left(\varepsilon_{\varphi M}\right)_{x_n = x_E}$, are functions of $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E}$. The tangential strain $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E}$ includes the integration constant C_e, which is determined by Equation (2.33). The tangential strain $\left(\varepsilon_{\varphi M}\right)_{x_n = x_E}$ is a function of the normal stress p_{1n} and the coefficient ρ_{2ME} (see Equations (2.27), (3.53), (4.130), (6.135)).

As analysed in [2]–[23], the following additional boundary condition can be considered

$$\left(\varepsilon_{nM}\right)_{x_n = x_M} = 0. \tag{2.32}$$

2.4.2 Envelope

If $\beta_{IN} \neq \beta_E \neq \beta_M$, then the boundary conditions have the forms [2]–[23]

$$\left(\sigma_{nE}\right)_{x_n = x_{IN}} = -p_{1n}, \tag{2.33}$$

$$\left(\sigma_{nE}\right)_{x_n = x_E} = -p_{2n}. \tag{2.34}$$

If $\beta_{IN} \neq \beta_E = \beta_M$, then $|u_{nE}| = f(x_n)$ represents a decreasing function of $x_n \in \langle x_{IN}, x_E \rangle$ (see Equation (1.22)). Consequently, the integration constant C_e is determined by Equation (2.33) [2]–[23].

If $\beta_{IN} = \beta_E \neq \beta_M$, then $|u_{nE}| = f(x_n)$ represents an increasing function of $x_n \in \langle x_{IN}, x_E \rangle$ (see Equation (1.22)). Consequently, the integration constant C_e is determined by Equation (2.34) [2]–[23].

2.4.3 Inclusion

The absolute value $|u_{nE}| = f(x_n)$ represents an increasing function of $x_n \in \langle 0, x_{IN} \rangle$ with a maximum value on the envelope-inclusion boundary, i.e., for $x_n = x_{IN}$. Additionally, the conditions $\left(u_{nIN}\right)_{x_n \to 0} \not\longrightarrow \pm\infty$, $\left(\varepsilon_{IN}\right)_{x_n \longrightarrow 0} \not\longrightarrow \pm\infty$, $\left(\sigma_{IN}\right)_{r \to 0} \not\longrightarrow \pm\infty$ are required to be fulfilled [2]–[23].

If $\beta_{IN} \neq \beta_E \neq \beta_M$ or $\beta_{IN} \neq \beta_E = \beta_M$, then the boundary conditions have the forms [2]–[23]

$$\left(u_n\right)_{x_n = 0} = 0, \tag{2.35}$$

$$\left(\sigma_{nIN}\right)_{x_n=x_{IN}} = -p_{1n}, \tag{2.36}$$

where x_{IN} is given by Equation (1.22), and the integration constant C_{IN} is determined by Equation (2.36). Equations (2.35) and (2.36) represent geometric and stress boundary conditions, respectively. If $\beta_{IN} \neq \beta_E \neq \beta_M$ or $\beta_{IN} \neq \beta_E = \beta_M$, then the normal stress p_{1n}, which acts on the envelope-inclusion boundary, is given by Equations (2.25) or (2.27) [2]–[23], respectively.

If $\beta_{IN} = \beta_E \neq \beta_M$, then we get $\left(u_{nIN}\right)_{x_n=x_{IN}} = \left(u_{nE}\right)_{x_n=x_{IN}}$. With regard to Equation (2.2), we get $\left(u_{nIN}\right)_{x_n=x_{IN}} = x_{IN}\left(\varepsilon_{\varphi IN}\right)_{x_n=x_{IN}}$, $\left(u_{nE}\right)_{x_n=x_{IN}} = x_{IN}\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}}$. Consequently, Equation 2.36 is replaced by the following mathematical boundary condition [2]–[23]

$$\left(\varepsilon_{\varphi IN}\right)_{x_n=x_{IN}} = \left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}} = -p_{2n}\rho_{1INE}, \tag{2.37}$$

where ρ_{1INE} is given by Equations (3.63), (4.140), (5.129), (5.139), (6.143) with respect to minimum W_c (see Equation (2.23)). In this case, the integration constant C_p in $\left(\varepsilon_{\varphi IN}\right)_{x_n=x_{IN}}$ is a function of $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}}$ [2]–[23]. The tangential strain $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}}$ includes the integration constant C_e, which is determined by Equation (2.34). The tangential strain $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}}$ is a function of the normal stress p_{2n} and the coefficient ρ_{1INE} (see Equation (3.63)). The normal stress p_{2n}, which acts on the envelope-matrix boundary, is given by Equation (2.28).

Chapter 3

Mathematical Model 1

3.1 Mathematical Procedure

Let the mathematical procedure $x_n\,[\partial \mathrm{Eq.}(2.20)/\partial x_n]$ be performed, and then we get [2]–[23]

$$x_n^2\,\frac{\partial^2 U_n}{\partial x_n^2}+(1-c_3)\,x_n\,\frac{\partial U_n}{\partial x_n}=0, \tag{3.1}$$

where $c_3<0$ and $U_n=U_n\,(x_n,\varphi,\theta)$ are given by Equations (2.18) and (2.21), respectively. Let Equation (2.20) be substituted to Equation (3.1), and then we get [2]–[23]

$$x_n^2\,\frac{\partial^2 U_n}{\partial x_n^2}+c_3\,(1-c_3)\,U_n=0. \tag{3.2}$$

Let U_n be assumed in the form $U_n=x_n^{\lambda}$, then we get [2]–[23]

$$U_n=C_1\,x_n^{\lambda_1}+C_2\,x_n^{\lambda_2}, \tag{3.3}$$

where C_1, C_2 are integration constants, which are determined by the boundary conditions in Section 2.4, and λ_1, λ_2, with respect to $\mu<0.5$ for a real isotropic material [26], have the forms [2]–[23]

$$\begin{aligned}\lambda_1&=\frac{1}{2}\left[1+\sqrt{1+16\,(1-\mu)\,[1+4\,(1-\mu)]}\right]>3,\\ \lambda_2&=\frac{1}{2}\left[1-\sqrt{1+16\,(1-\mu)\,[1+4\,(1-\mu)]}\right]<-2.\end{aligned} \tag{3.4}$$

Let Equation (3.3) be substituted to Equation (2.19), and then we get [2]–[23]

$$x_n^2\,\frac{\partial^2 u_n}{\partial x_n^2}+2\,x_n\,\frac{\partial u_n}{\partial x_n}-2\,u_n=C_1\,x_n^{\lambda_1}+C_2\,x_n^{\lambda_2}. \tag{3.5}$$

The mathematical solution of Equation (3.5), which is determined by Wronskian's method [24], is derived as

$$u_n = \sum_{i=1}^{2} C_i x_n^{\lambda_i}. \tag{3.6}$$

With regard to Equations (2.1)–(2.4), (2.14)–(2.17), (2.22), (3.6), we get

$$\varepsilon_n = \sum_{i=1}^{2} C_i \lambda_i x_n^{\lambda_i - 1}, \tag{3.7}$$

$$\varepsilon_\varphi = \sum_{i=1}^{2} C_i x_n^{\lambda_i - 1}, \tag{3.8}$$

$$\varepsilon_{n\varphi} = s_{44}\,\sigma_{n\varphi} = \sum_{i=1}^{2} \frac{\partial C_i}{\partial \varphi} x_n^{\lambda_i - 1}, \tag{3.9}$$

$$\varepsilon_{n\theta} = s_{44}\,\sigma_{n\theta} = \Theta \sum_{i=1}^{2} \frac{\partial C_i}{\partial \nu} x_n^{\lambda_i - 1}, \tag{3.10}$$

$$\sigma_n = \sum_{i=1}^{2} C_i \xi_i x_n^{\lambda_i - 1}, \tag{3.11}$$

$$\sigma_\varphi = \sigma_\theta = \sum_{i=1}^{2} C_i \xi_{2+i} x_n^{\lambda_i - 1}, \tag{3.12}$$

$$w = \kappa_1 x_n^{2(\lambda_1 - 1)} + \kappa_2 x_n^{2(\lambda_2 - 1)} + +\kappa_3 x_n^{\lambda_1 + \lambda_2 - 2}, \tag{3.13}$$

where Θ, s_{44} is given by Equations (1.21), (2.13), respectively, and ξ_i, ξ_{2+i}, ξ_{2+i+2j}, κ_j $(i = 1,2;\ j = 1,2,3)$ are derived as

$$\begin{aligned}
&\xi_i = \frac{E\left[\lambda_i (1-\mu) + 2\mu\right]}{(1+\mu)(1-2\mu)}, \quad \xi_{2+i} = \frac{E\,(1+\lambda_i \mu)}{(1+\mu)(1-2\mu)}, \\
&\xi_{2+i+2j} = \frac{E\left\{\lambda_i\left[\lambda_j (1-\mu) + 4\mu\right] + 2\right\}}{2(1+\mu)(1-2\mu)}, \\
&\kappa_i = C_i^2 \xi_{2+3i} + \frac{1}{s_{44}}\left[\left(\frac{\partial C_i}{\partial \varphi}\right)^2 + \Theta^2 \left(\frac{\partial C_i}{\partial \nu}\right)^2\right], \\
&\kappa_3 = C_1 C_2 (\xi_6 + \xi_7) + \frac{1}{s_{44}}\left(\frac{\partial C_1}{\partial \varphi}\frac{\partial C_2}{\partial \varphi} + \Theta^2 \frac{\partial C_1}{\partial \nu}\frac{\partial C_2}{\partial \nu}\right), \quad i, j = 1,2.
\end{aligned} \tag{3.14}$$

In case of the thermal stresses in the ellipsoidal inclusion, $|u_{nIN}| = f(x_n)$ represents an increasing function of $x_n \in \langle 0, x_{IN} \rangle$, and then $C_{1IN} \neq 0$, $C_{2IN} = 0$ (see Equation (3.6)). If $C_{1IN} \neq 0$, $C_{2IN} = 0$, then the thermal stresses in the ellipsoidal inclusion are determined in [1]. The radial stress p, which acts on on the inclusion-matrix boundary is replaced by p_{1n} (see Equations (2.25), (2.27)), where ρ_{IN} in Equations (2.25), (2.27) is determined in [1].

3.2 Condition $\beta_{IN} \neq \beta_E \neq \beta_M$

3.2.1 Matrix

With regard to Equations (2.29), (2.30), (3.6)–(3.13), (2.22), (2.23), we get

$$\varepsilon_{nM} = -p_{2n} \sum_{i=1}^{2} \frac{\lambda_{iM}}{\zeta_{iM}} \left(\frac{x_n}{x_M} \right)^{\lambda_{iM}-1}, \tag{3.15}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -p_{2n} \sum_{i=1}^{2} \frac{1}{\zeta_{iM}} \left(\frac{x_n}{x_M} \right)^{\lambda_{iM}-1}, \tag{3.16}$$

$$\varepsilon_{n\varphi M} = s_{44M}\, \sigma_{n\varphi M} = -\frac{\partial}{\partial \varphi} \sum_{i=1}^{2} \left(\frac{p_{2n}}{\zeta_{iM}\, x_M^{\lambda_{iM}-1}} \right) x_n^{\lambda_{iM}-1}, \tag{3.17}$$

$$\varepsilon_{n\theta M} = s_{44M}\, \sigma_{n\theta M} = -\Theta \sum_{i=1}^{2} \frac{\partial}{\partial \nu} \left(\frac{p_{2n}}{\zeta_{iM}\, x_M^{\lambda_{iM}-1}} \right) x_n^{\lambda_{iM}-1}, \tag{3.18}$$

$$\sigma_{nM} = -p_{2n} \sum_{i=1}^{2} \frac{\xi_{iM}}{\zeta_{iM}} \left(\frac{x_n}{x_M} \right)^{\lambda_{iM}-1}, \tag{3.19}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -p_{2n} \sum_{i=1}^{2} \frac{\xi_{2+iM}}{\zeta_{iM}} \left(\frac{x_n}{x_M} \right)^{\lambda_{iM}-1}, \tag{3.20}$$

$$w_M = \kappa_{1M}\, x_n^{2(\lambda_{1M}-1)} + \kappa_{2M}\, x_n^{2(\lambda_{2M}-1)} + + \kappa_{3M}\, x_n^{\lambda_{1M}+\lambda_{2M}-2}, \tag{3.21}$$

$$W_M = 4 \int_0^{\pi/2} \int_0^{\pi/2} \left[\frac{\kappa_{1M} \left(x_M^{2\lambda_{1M}+1} - x_E^{2\lambda_{1M}+1} \right)}{2\lambda_{1M}+1} + \frac{\kappa_{2M} \left(x_M^{2\lambda_{2M}+1} - x_E^{2\lambda_{2M}+1} \right)}{2\lambda_{2M}+1} + \frac{\kappa_{3M} \left(x_M^{\lambda_{1M}+\lambda_{2M}+1} - x_E^{\lambda_{1M}+\lambda_{2M}+1} \right)}{\lambda_{1M}+\lambda_{2M}+1} \right] d\varphi\, d\nu, \tag{3.22}$$

where Θ, x_E, x_M, s_{44M}, λ_{iM}, ξ_{jM} ($i = 1,2;\ j = 1, \ldots, 4$) are given by Equations (1.21), (1.22), (2.13), (3.4), (3.14), respectively, and ζ_{iM}, κ_{jM} ($i = 1,2;\ j = 1,2,3$; see Equation (3.14)) have the forms

$$
\begin{aligned}
\zeta_{iM} &= \xi_{iM}\left(\frac{x_E}{x_M}\right)^{\lambda_{iM}-1} - \xi_{3-iM}\left(\frac{x_E}{x_M}\right)^{\lambda_{3-iM}-1}, \\
\kappa_{iM} &= \xi_{2+3iM}\left(\frac{p_{2n}}{\zeta_{iM}\,x_M^{\lambda_{iM}-1}}\right)^2 + \frac{1}{s_{44M}}\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_{iM}\,x_M^{\lambda_{iM}-1}}\right)\right]^2 \\
&\quad + \frac{\Theta^2}{s_{44M}}\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_{iM}\,x_M^{\lambda_{iM}-1}}\right)\right]^2, \\
\kappa_{3M} &= \frac{p_{2n}^2\left(\xi_{6M}+\xi_{7M}\right)}{\zeta_1\,\zeta_2\,x_M^{\lambda_{1M}+\lambda_{2M}-2}} + \frac{1}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_1\,x_M^{\lambda_{1M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_2\,x_M^{\lambda_{2M}-1}}\right) \\
&\quad + \frac{\Theta^2}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_1\,x_M^{\lambda_{1M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_2\,x_M^{\lambda_{2M}-1}}\right), \quad i=1,2.
\end{aligned}
\tag{3.23}
$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i, j=1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E} = -p_{2n}\,\rho_M$ and Equation (3.16), the coefficient ρ_M in Equation (2.26) is derived as

$$
\rho_M = \frac{(1+\mu)(1-2\mu)}{E}\left[\frac{1}{\lambda_1(1-\mu)+2\mu} + \frac{1}{\lambda_2(1-\mu)+2\mu}\right]. \tag{3.24}
$$

3.2.2 Envelope

With regard to Equations (2.33), (2.34), (3.6)–(3.13), (2.22), (2.23), we get

$$
\varepsilon_{nE} = -\frac{1}{\zeta_E}\sum_{i=1}^{2}\frac{\lambda_{iE}}{\xi_{iE}}\left[p_{1n} - p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1}\right]\left(\frac{x_n}{x_{IN}}\right)^{\lambda_{iE}-1}, \tag{3.25}
$$

$$
\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\frac{1}{\zeta_E}\sum_{i=1}^{2}\frac{1}{\xi_{iE}}\left[p_{1n} - p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1}\right]\left(\frac{x_n}{x_{IN}}\right)^{\lambda_{iE}-1}, \tag{3.26}
$$

$$
\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\sum_{i=1}^{2}\frac{x_n^{\lambda_{iE}-1}}{\xi_{iE}}\frac{\partial}{\partial\varphi}\left\{\frac{1}{\zeta_E\,x_{IN}^{\lambda_{iE}-1}}\left[p_{1n} - p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1}\right]\right\}, \tag{3.27}
$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\Theta\,\sum_{i=1}^{2}\frac{x_n^{\lambda_{iE}-1}}{\xi_{iE}}\,\frac{\partial}{\partial\nu}\left\{\frac{1}{\zeta_E\,x_{IN}^{\lambda_{iE}-1}}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1}\right]\right\}, \tag{3.28}$$

$$\sigma_{nE} = -\,\frac{1}{\zeta_E}\sum_{i=1}^{2}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1}\right]\left(\frac{x_n}{x_{IN}}\right)^{\lambda_{iE}-1}, \tag{3.29}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\,\frac{1}{\zeta_E}\sum_{i=1}^{2}\frac{\xi_{2+iE}}{\xi_{iE}}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1}\right]\left(\frac{x_n}{x_{IN}}\right)^{\lambda_{iE}-1}, \tag{3.30}$$

$$w_E = \kappa_{1E}\,x_n^{2(\lambda_{1E}-1)} + \kappa_{2E}\,x_n^{2(\lambda_{2E}-1)} + +\kappa_{3E}\,x_n^{\lambda_{1E}+\lambda_{2E}-2}, \tag{3.31}$$

$$\begin{aligned} W_E = 4\int_0^{\pi/2}\!\!\int_0^{\pi/2}\Bigg[&\frac{\kappa_{1E}\left(x_E^{2\lambda_{1E}+1}-x_{IN}^{2\lambda_{1E}+1}\right)}{2\lambda_{1E}+1}+\frac{\kappa_{2E}\left(x_E^{2\lambda_{2E}+1}-x_{IN}^{2\lambda_{2E}+1}\right)}{2\lambda_{2E}+1}\\ &+\frac{\kappa_{3E}\left(x_E^{\lambda_{1E}+\lambda_{2E}+1}-x_{IN}^{\lambda_{1E}+\lambda_{2E}+1}\right)}{\lambda_{1E}+\lambda_{2E}+1}\Bigg]\,d\varphi\,d\nu, \end{aligned} \tag{3.32}$$

where Θ, x_{IN}, x_E, s_{44E}, λ_{iE}, ξ_{jE} $(i=1,2;\ j=1,\ldots,7)$ are given by Equations (1.21), (1.22), (2.13), (3.4), (3.14), respectively, and ζ_E, κ_{iE} $(i=1,2,3$; see Equation (3.14)) have the forms

$$\zeta_E = 1-\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{2E}-\lambda_{1E}},$$

$$\begin{aligned} \kappa_{iE} = {} & \frac{\xi_{2+3i}}{\left(\xi_{ie}\,\zeta_e\,x_{IN}^{\lambda_{iE}-1}\right)^2}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1}\right]^2\\ & + \frac{1}{s_{44E}\,\xi_{ie}^2}\left(\frac{\partial}{\partial\varphi}\left\{\frac{1}{\zeta_e x_{IN}^{\lambda_{iE}-1}}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1}\right]\right\}\right)^2\\ & + \frac{\Theta^2}{s_{44E}}\left(\frac{\partial}{\partial\nu}\left\{\frac{1}{\zeta_e x_{IN}^{\lambda_{iE}-1}}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1}\right]\right\}\right)^2, \quad i=1,2, \end{aligned}$$

$$\kappa_{3E} = \frac{1}{\xi_{1E}\,\xi_{2E}\,\zeta_E^2\,x_{IN}^{\lambda_{1E}+\lambda_{2E}-2}}$$

$$\times\left\{(\xi_{6E}+\xi_{7E})\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{2E}-1}\right]\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{1E}-1}\right]\right.$$
$$+\frac{1}{s_{44E}}\frac{\partial}{\partial\varphi}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{2E}-1}\right]\frac{\partial}{\partial\varphi}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{1E}-1}\right]$$
$$\left.+\frac{\Theta^2}{s_{44E}}\frac{\partial}{\partial\nu}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{2E}-1}\right]\frac{\partial}{\partial\nu}\left[p_{1n}-p_{2n}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{1E}-1}\right]\right\}. \quad (3.33)$$

The normal stresses p_{1n}, p_{2n} are given by Equations (2.25), (2.26), where ρ_{IN}, ρ_M in Equation (2.25) are given by Equations (6.89), (3.24), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}}=-\left(p_{1n}\rho_{11E}+p_{2n}\rho_{12E}\right)$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E}=-\left(p_{1n}\rho_{21E}+p_{2n}\rho_{22E}\right)$ and Equation (3.26), the coefficient ρ_{ijE} $(i,j=1{,}2)$ in Equations (2.25), (2.26) is derived as

$$\rho_{11E}=\frac{1}{\zeta_E}\sum_{i=1}^{2}\frac{1}{\xi_{iE}},\quad \rho_{21E}=\frac{1}{\zeta_E}\sum_{i=1}^{2}\frac{1}{\xi_{iE}}\left(\frac{x_E}{x_{IN}}\right)^{\lambda_{iE}-1},$$
$$\rho_{12E}=-\frac{1}{\zeta_E}\sum_{i=1}^{2}\frac{1}{\xi_{iE}}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-iE}-1},$$
$$\rho_{22E}=-\frac{1}{\zeta_E}\sum_{i=1}^{2}\frac{1}{\xi_{iE}}\left(\frac{x_{IN}}{x_E}\right)^{\lambda_{3-ie}-1}\left(\frac{x_E}{x_{IN}}\right)^{\lambda_{iE}-1}. \quad (3.34)$$

3.3 Condition $\beta_{IN}\neq\beta_E=\beta_M$

3.3.1 Matrix

With regard to Equations (2.30), (2.31), (3.6)–(3.13), (2.22), (2.23), we get

$$\varepsilon_{nM}=-p_{1n}\rho_{2ME}\sum_{i=1}^{2}\frac{\lambda_{iM}}{\zeta_{iM}}\left(\frac{x_n}{x_M}\right)^{\lambda_{iM}-1}, \quad (3.35)$$

$$\varepsilon_{\varphi M}=\varepsilon_{\theta M}=-p_{1n}\rho_{2ME}\sum_{i=1}^{2}\frac{1}{\zeta_{iM}}\left(\frac{x_n}{x_M}\right)^{\lambda_{iM}-1}, \quad (3.36)$$

$$\varepsilon_{n\varphi M}=s_{44M}\sigma_{n\varphi M}=\sum_{i=1}^{2}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_{iM}x_M^{\lambda_{iM}-1}}\right)\left(\frac{x_n}{x_M}\right)^{\lambda_{iM}-1}, \quad (3.37)$$

$$\varepsilon_{n\theta M}=s_{44M}\sigma_{n\theta M}=-\Theta\sum_{i=1}^{2}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_{iM}x_M^{\lambda_{iM}-1}}\right)\left(\frac{x_n}{x_M}\right)^{\lambda_{iM}-1}, \quad (3.38)$$

$$\sigma_{nM} = -p_{1n}\rho_{2ME}\sum_{i=1}^{2}\frac{\xi_{iM}}{\zeta_{iM}}\left(\frac{x_n}{x_M}\right)^{\lambda_{iM}-1}, \tag{3.39}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -p_{1n}\rho_{2ME}\sum_{i=1}^{2}\frac{\xi_{2+iM}}{\zeta_{iM}}\left(\frac{x_n}{x_M}\right)^{\lambda_{iM}-1}, \tag{3.40}$$

$$w_M = \kappa_{1M}x_n^{2(\lambda_{1M}-1)} + \kappa_{2M}x_n^{2(\lambda_{2M}-1)} + +\kappa_{3M}x_n^{\lambda_{1M}+\lambda_{2M}-2}, \tag{3.41}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{1M}\left(x_M^{2\lambda_{1M}+1} - x_E^{2\lambda_{1M}+1}\right)}{2\lambda_{1M}+1} + \frac{\kappa_{2M}\left(x_M^{2\lambda_{2M}+1} - x_E^{2\lambda_{2M}+1}\right)}{2\lambda_{2M}+1}\right.$$
$$\left. + \frac{\kappa_{3M}\left(x_M^{\lambda_{1M}+\lambda_{2M}+1} - x_E^{\lambda_{1M}+\lambda_{2M}+1}\right)}{\lambda_{1M}+\lambda_{2M}+1}\right] d\varphi\, d\nu, \tag{3.42}$$

where Θ, x_E, x_M, s_{44M}, λ_{iM}, ξ_{jM} $(i=1,2;\ j=1,\ldots,4)$ are given by Equations (1.21), (1.22), (2.13), (3.4), (3.14), respectively, and ζ_{iM}, κ_{jM} $(i=1,2;\ j=1,2,3$; see Equation (3.14)) have the forms

$$\zeta_{iM} = \left(\frac{x_E}{x_M}\right)^{\lambda_{iM}-1} - \left(\frac{x_E}{x_M}\right)^{\lambda_{3-iM}-1},$$
$$\kappa_{iM} = \xi_{2+3iM}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_{iM}x_M^{\lambda_{iM}-1}}\right)^2 + \frac{1}{s_{44M}}\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_{iM}x_M^{\lambda_{iM}-1}}\right)\right]^2$$
$$+ \frac{\Theta^2}{s_{44M}}\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_{iM}x_M^{\lambda_{iM}-1}}\right)\right]^2,$$
$$\kappa_{3M} = \frac{p_{1n}\rho_{2ME}^2(\xi_{6M}+\xi_{7M})}{\zeta_1\zeta_2 x_M^{\lambda_{1M}+\lambda_{2M}-2}} + \frac{1}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_1 x_M^{\lambda_{1M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_2 x_M^{\lambda_{2M}-1}}\right)$$
$$+ \frac{\Theta^2}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_1 x_M^{\lambda_{1M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_2 x_M^{\lambda_{2M}-1}}\right), \quad i=1,2. \tag{3.43}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{1ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively , with respect to minimum W_c (see Equation (2.23)).

3.3.2 Envelope

If $\beta_{IN} \neq \beta_E = \beta_M$, then $|u_{nE}| = f(x_n)$ represents a decreasing function of $x_n \in \langle x_{IN}, x_E \rangle$, and then $C_{1e} = 0$ and $C_{2e} \neq 0$ due to $\lambda_{1e} > 3$ and $\lambda_{2e} < -2$ (see Equations (3.4), (3.6)), respectively. With regard to Equations (2.33), (3.6)–(3.13), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\frac{p_{1n}\lambda_{2E}}{\xi_{2E}}\left(\frac{x_n}{x_{IN}}\right)^{\lambda_{2E}-1}, \tag{3.44}$$

$$\varepsilon_{\varphi E} = \varepsilon_{33E} = -\frac{p_{1n}}{\xi_{2E}}\left(\frac{x_n}{x_{IN}}\right)^{\lambda_{2E}-1}, \tag{3.45}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\xi_{2E}\,x_{IN}^{\lambda_{2E}-1}}\right)x_n^{\lambda_{2E}-1}, \tag{3.46}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\Theta\,\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\xi_{2E}\,x_{IN}^{\lambda_{2E}-1}}\right)x_n^{\lambda_{2E}-1}, \tag{3.47}$$

$$\sigma_{nE} = -p_{1n}\left(\frac{x_n}{x_{IN}}\right)^{\lambda_{2E}-1}, \tag{3.48}$$

$$\sigma_{\varphi E} = \sigma_{33E} = -\frac{p_{1n}\xi_{3E}}{\xi_{2E}}\left(\frac{x_n}{x_{IN}}\right)^{\lambda_{2E}-1}, \tag{3.49}$$

$$w_E = \kappa_{3E}\,x_n^{2(\lambda_{2E}-1)}, \tag{3.50}$$

$$W_E = \frac{4}{2\lambda_{2E}+1}\int_0^{\pi/2}\int_0^{\pi/2}\kappa_{3E}\left(x_E^{2\lambda_{2E}+1} - x_{IN}^{2\lambda_{2E}+1}\right)d\varphi\,d\nu, \tag{3.51}$$

where Θ, x_{IN}, x_E, s_{44E}, λ_{2E} and ξ_{2E}, ξ_{3E} are given by Equations (1.21), (1.22), (2.13), (3.4) and (3.14), respectively, and κ_{3E} (see Equation (3.14)) has the form

$$\begin{aligned}\kappa_{3E} = {}& \xi_{5E}\left(\frac{p_{1n}}{\xi_{2E}\,x_{IN}^{\lambda_{2E}-1}}\right)^2 \\ &+ \frac{1}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\xi_{2E}\,x_{IN}^{\lambda_{2E}-1}}\right)\right]^2 + \Theta^2\left[\frac{\partial C_i}{\partial\nu}\left(\frac{p_{1n}}{\xi_{2E}\,x_{IN}^{\lambda_{2E}-1}}\right)\right]^2\right\}.\end{aligned} \tag{3.52}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} in Equation (2.27) is given by Equation (6.89). With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}} = -p_{1n}\rho_{1ME}$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E} =$

$-p_{1n}\rho_{2ME}$ (see Equation (3.45)), the coefficients ρ_{1ME}, ρ_{2ME} in Equations (2.27), (2.31) are derived as

$$\rho_{1ME} = \frac{1}{\xi_{2E}}, \quad \rho_{2ME} = \frac{1}{\xi_{2E}} \left(\frac{x_E}{x_{IN}} \right)^{\lambda_{2E}-1}. \tag{3.53}$$

3.4 Condition $\beta_{IN} = \beta_E \neq \beta_M$

If $\beta_{IN} = \beta_E \neq \beta_M$, then the thermal strains and stresses in the cell matrix are determined in Section 3.2.1. The normal stress p_{2n} in Section 3.2.1 is given by Equation (2.28). The coefficients ρ_{IN}; ρ_M; ρ_{2INE} in Equation (2.28) are given by Equations (6.89); (3.24), (4.24), (4.34), (4.44), (4.54), (5.23), (5.33), (5.43), (5.53), (6.22), (6.32), (6.42), (6.52), (7.23), (7.33), (7.43), (7.53), (7.63), (7.73); (3.63), respectively, with respect to minimum W_c (see Equation (2.23)).

3.4.1 Envelope

If $\beta_{IN} = \beta_E \neq \beta_M$, then $|u_{nE}| = f(x_n)$ represents an increasing function of $x_n \in \langle x_{IN}, x_E \rangle$, and then $C_{1E} \neq 0$ and $C_{2E} = 0$ due to $\lambda_{1e} > 3$ and $\lambda_{2E} < -2$ (see Equations (3.4), (3.6)), respectively. With regard to Equations (2.34), (3.6)–(3.13), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\frac{p_{2n}\lambda_{1E}}{\xi_{1E}} \left(\frac{x_n}{x_E} \right)^{\lambda_{1E}-1}, \tag{3.54}$$

$$\varepsilon_{\varphi E} = \varepsilon_{33E} = -\frac{p_{2n}}{\xi_{1E}} \left(\frac{x_n}{x_E} \right)^{\lambda_{1E}-1}, \tag{3.55}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\frac{\partial}{\partial\varphi} \left(\frac{p_{2n}}{\xi_{1E}\,x_E^{\lambda_{1E}-1}} \right) x_n^{\lambda_{1E}-1}, \tag{3.56}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\Theta\,\frac{\partial}{\partial\nu} \left(\frac{p_{2n}}{\xi_{1E}\,x_E^{\lambda_{1E}-1}} \right) x_n^{\lambda_{1E}-1}, \tag{3.57}$$

$$\sigma_{nE} = -p_{2n} \left(\frac{x_n}{x_E} \right)^{\lambda_{1E}-1}, \tag{3.58}$$

$$\sigma_{\varphi E} = \sigma_{33E} = -\frac{p_{2n}\xi_{3E}}{\xi_{1E}} \left(\frac{x_n}{x_E} \right)^{\lambda_{1E}-1}, \tag{3.59}$$

$$w_E = \kappa_{1E}\,x_n^{2(\lambda_{1E}-1)}, \tag{3.60}$$

$$W_E = \frac{4}{2\lambda_{1E}+1} \int\limits_0^{\pi/2} \int\limits_0^{\pi/2} \kappa_{1E} \left(x_E^{2\lambda_{1E}+1} - x_{IN}^{2\lambda_{1E}+1} \right) d\varphi \, d\nu, \tag{3.61}$$

where Θ, x_{IN}, x_E, s_{44E}, λ_{1E} and ξ_{1E}, ξ_{3E} are given by Equations (1.21), (1.22), (2.13), (3.4) and (3.14), respectively, and κ_{1E} (see Equation (3.14)) has the form

$$\kappa_{1E} = \xi_{5E} \left(\frac{p_{2n}}{\xi_{1E} x_E^{\lambda_{1E}-1}} \right)^2 + \frac{1}{s_{44E}} \left\{ \left[\frac{\partial}{\partial\varphi} \left(\frac{p_{2n}}{\xi_{1E} x_E^{\lambda_{1E}-1}} \right) \right]^2 + \Theta^2 \left[\frac{\partial C_i}{\partial\nu} \left(\frac{p_{2n}}{\xi_{1E} x_E^{\lambda_{1E}-1}} \right) \right]^2 \right\}. \tag{3.62}$$

The normal stress p_{2n} is given by Equation (2.28), where ρ_M in Equation (2.28) is given by Equations (3.24), (4.24), (4.34), (4.44), (4.54), (5.23), (5.33), (5.43), (5.53), (6.22), (6.32), (6.42), (6.52), (7.23), (7.33), (7.43), (7.53), (7.63), (7.73) with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n = x_{IN}} = -p_{2n}\rho_{1INE}$, $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E} = -p_{2n}\rho_{2INE}$ (see Equation (3.55)), the coefficients ρ_{1INE}, ρ_{2INE} in Equations (2.28), (2.37) are derived as

$$\rho_{1INE} = \frac{1}{\xi_{1E}} \left(\frac{x_{IN}}{x_E} \right)^{\lambda_{1E}-1}, \quad \rho_{2INE} = \frac{1}{\xi_{1E}}. \tag{3.63}$$

Chapter 4

Mathematical Model 2

4.1 Mathematical Procedure

Let the mathematical procedure $\partial^2\text{Eq.}(2.20)/\partial x_n^2$ be performed, and then we get [2]–[23]

$$x_n \frac{\partial^3 U_n}{\partial x_n^3} + (2 - c_3) \frac{\partial^2 U_n}{\partial x_n^2} = 0, \tag{4.1}$$

where $c_3 < 0$ and $U_n = U_n(x_n, \varphi, \nu)$ are given by Equations (2.18) and (2.21), respectively. Let U_b be assumed in the form $U_n = x_n^\lambda$, and then we get

$$U_n = C_1 x_n + C_2 x_n^{c_3} + C_3, \tag{4.2}$$

where C_1, C_2, C_3 are integration constants, which are determined by the boundary conditions in Section 2.4. Let Equation (2.35) be substituted to Equation (2.19), and then we get

$$x_n^2 \frac{\partial^2 u_n}{\partial x_n^2} + 2x_n \frac{\partial u_n}{\partial x_n} - 2u_n = C_1 x_n + C_2 x_n^{c_3} + C_3 x_n^2. \tag{4.3}$$

The mathematical solution of Equation (4.3), which is determined by Wronskian's method [24], is derived as

$$u_n = C_1 x_n \left(\frac{1}{3} - \ln x_n\right) + C_2 x_n^{c_3} + C_3. \tag{4.4}$$

With regard to Equations (2.1)–(2.4), (2.14)–(2.17), (2.22), (4.4), we get

$$\varepsilon_n = -C_1 \left(\frac{2}{3} + \ln x_n\right) + C_2 c_3 x_n^{c_3 - 1}, \tag{4.5}$$

$$\varepsilon_\varphi = \varepsilon_\theta = C_1 \left(\frac{1}{3} - \ln x_n\right) + C_2 x_n^{c_3 - 1} + \frac{C_3}{x_n}, \tag{4.6}$$

$$\varepsilon_{n\varphi} = s_{44} \sigma_{n\varphi} = \left(\frac{1}{3} - \ln x_n\right) \frac{\partial C_1}{\partial \varphi} + x_n^{c_3 - 1} \frac{\partial C_2}{\partial \varphi} + \frac{1}{x_n} \frac{\partial C_3}{\partial \varphi}, \tag{4.7}$$

$$\varepsilon_{n\theta} = s_{44}\,\sigma_{n\theta} = \Theta\left[\left(\frac{1}{3} - \ln x_n\right)\frac{\partial C_1}{\partial \nu} + x_n^{c_3-1}\frac{\partial C_2}{\partial \nu} + \frac{1}{x_n}\frac{\partial C_3}{\partial \nu}\right], \tag{4.8}$$

$$\sigma_n = -\,C_1\left[\frac{2\,(c_1+2\,c_2)}{3} + (c_1-c_2)\ln x_n\right] + C_2\left[(c_1+c_2)\,c_3 - 2\,c_2\right]x_n^{c_3-1} - \frac{2\,C_3\,c_2}{x_n}, \tag{4.9}$$

$$\sigma_\varphi = \sigma_\theta = C_1\left[\frac{c_1+2\,c_2}{3} - (c_1-c_2)\ln x_n\right] + C_2\,(c_1 - c_2\,c_3)\,x_n^{c_3-1} + \frac{C_3\,c_1}{x_n}, \tag{4.10}$$

$$\begin{aligned} w = {} & C_1^2\,\kappa_1 + C_2^2\,\kappa_2 + C_3^2\,\kappa_3 + C_1\,C_2\,\kappa_4 + C_1\,C_3\,\kappa_5 + C_2\,C_3\,\kappa_6 \\ & + \frac{\chi_1}{s_{44}}\left[\left(\frac{\partial C_1}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial C_1}{\partial \nu}\right)^2\right] + \frac{\chi_2}{s_{44}}\left[\left(\frac{\partial C_2}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial C_2}{\partial \nu}\right)^2\right] \\ & + \frac{\chi_3}{s_{44}}\left[\left(\frac{\partial C_3}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial C_3}{\partial \nu}\right)^2\right] + \frac{\chi_4}{s_{44}}\left(\frac{\partial C_1}{\partial \varphi}\frac{\partial C_2}{\partial \varphi} + \Theta^2\frac{\partial C_1}{\partial \nu}\frac{\partial C_2}{\partial \nu}\right) \\ & + \frac{\chi_5}{s_{44}}\left(\frac{\partial C_1}{\partial \varphi}\frac{\partial C_3}{\partial \varphi} + \Theta^2\frac{\partial C_1}{\partial \nu}\frac{\partial C_3}{\partial \nu}\right) + \frac{\chi_6}{s_{44}}\left(\frac{\partial C_2}{\partial \varphi}\frac{\partial C_3}{\partial \varphi} + \Theta^2\frac{\partial C_2}{\partial \nu}\frac{\partial C_3}{\partial \nu}\right), \end{aligned} \tag{4.11}$$

where Θ, c_i $(i=1,2,3)$, s_{44} are given by Equations (1.21), (2.18), (2.13), respectively, and κ_j, χ_j $(j=1,\ldots,6)$ are derived as

$$\begin{aligned} \kappa_1 &= \frac{c_2-c_1}{2}\ln^2 x_n + \frac{2\,(c_2-c_1)}{3}\ln x_n + \frac{7c_1+2\,c_2}{9}, \\ \kappa_2 &= \left[\frac{c_3^2\,(c_1+c_2)}{2} + c_1\,(1-2\,c_3)\right]x_n^{2(c_3-1)}, \quad \kappa_3 = \frac{c_1}{x_n^2}, \\ \kappa_4 &= c_3\,(c_1-c_2)\,x_n^{c_3-1}\ln x_n + 2\left[c_1 - \frac{c_3\,(2\,c_1+c_2)}{3}\right]x_n^{c_3-1}, \\ \kappa_5 &= \frac{2\,c_1}{x_n}, \quad \kappa_6 = 0, \\ \chi_1 &= \ln^2 x_n - \frac{2}{3}\ln x_n + \frac{1}{9}, \quad \chi_2 = x_n^{2(c_3-1)}, \quad \chi_3 = \frac{1}{x_n^2}, \\ \chi_4 &= \frac{2}{3}x_n^{c_3-1} - 2x_n^{c_3-1}\ln x_n, \quad \chi_5 = \frac{2}{3x_n} - \frac{2\ln x_n}{x_n}, \quad \chi_6 = x_n^{c_3-2}. \end{aligned} \tag{4.12}$$

The integrals Φ_i, Ψ_i of the $\kappa_j = \kappa_j\,(x_n)$, $\chi_j = \chi_j\,(x_n)$ $(i=1,\ldots,6)$, respectively, have the forms

$$\Phi_{iq} = \int\limits_{x_{1n}}^{x_{2n}} \kappa_{iq} x_n^2 \, dx_n, \quad \Psi_{iq} = \int\limits_{x_{1n}}^{x_{2n}} \chi_{iq} x_n^2 \, dx_n, \quad i = 1, \ldots, 6,$$

$$q = E \;\; \Rightarrow \;\; x_{1n} = x_{IN}, \; x_{2n} = x_E,$$

$$q = M \;\; \Rightarrow \;\; x_{1n} = x_E, \; x_{2n} = x_M. \tag{4.13}$$

where x_{IN}, x_E, x_M are given by Equations (1.22), respectively. Consequently, we get

$$\begin{aligned}
\Phi_{1q} &= \frac{c_{2q} - c_{1q}}{6} \left\{ x_{2n}^3 \left[\left(\ln x_{2n} - \frac{1}{3} \right)^2 + \frac{1}{9} \right] - x_{1n}^3 \left[\left(\ln x_{1n} - \frac{1}{3} \right)^2 + \frac{1}{9} \right] \right\} \\
&+ \frac{2\left(c_{2q} - c_{1q}\right)}{9} \left[x_{2n}^3 \left(\ln x_{2n} - \frac{1}{3} \right) - x_{1n}^3 \left(\ln x_{1n} - \frac{1}{3} \right) \right] \\
&+ \frac{\left(7c_{1q} + 2\,c_{2q}\right)\left(x_{2n}^3 - x_{1n}^3\right)}{27}, \\
\Phi_{2q} &= \frac{1}{2\,c_{3q} + 1} \left[\frac{c_{3q}^2 \left(c_{1q} + c_{2q}\right)}{2} + c_{1q} \left(1 - 2\,c_{3q}\right) \right] \left(x_{2n}^{2\,c_{3q}+1} - x_{1n}^{2\,c_{3q}+1} \right), \\
\Phi_{3q} &= c_{1q} \left(x_{2n} - x_{1n}\right), \\
\Phi_{4q} &= \frac{c_{3q}\left(c_{1q} - c_{2q}\right)}{c_{3q} + 2} \left[x_{2n}^{c_{3q}+2} \left(\ln x_{2n} - \frac{1}{c_{3q} + 2} \right) - x_{1n}^{c_{3q}+2} \left(\ln x_{1n} - \frac{1}{c_{3q} + 2} \right) \right] \\
&+ \frac{2}{c_{3q} + 2} \left[c_{1q} - \frac{c_{3q}\left(2\,c_{1q} + c_{2q}\right)}{3} \right] \left(x_{2n}^{c_{3q}+2} - x_{1n}^{c_{3q}+2} \right), \\
\Phi_{5q} &= c_{1q} \left(x_{2n}^2 - x_{1n}^2 \right), \; \Phi_{6q} = 0, \\
\Psi_{1q} &= \frac{x_{2n}^3}{3} \left[\left(\ln x_{2n} - 1\right) \left(\ln x_{2n} - \frac{1}{3} \right) + \frac{2}{9} \right] - \frac{x_{1n}^3}{3} \left[\left(\ln x_{1n} - 1\right) \left(\ln x_{1n} - \frac{1}{3} \right) + \frac{2}{9} \right], \\
\Psi_{2q} &= \frac{x_{2n}^{2\,c_{3q}+1} - x_{1n}^{2\,c_{3q}+1}}{2\,c_{3q} + 1}, \quad \Psi_{3q} = x_{2n} - x_{1n}, \\
\Psi_{4q} &= \frac{2}{c_{3q} + 2} \left\{ x_{2n}^{c_{3q}+2} \left[\frac{c_{3q} + 5}{3\left(c_{3q} + 2\right)} - \ln x_{2n} \right] - x_{1n}^{c_{3q}+2} \left[\frac{c_{3q} + 5}{3\left(c_{3q} + 2\right)} - \ln x_{1n} \right] \right\}, \\
\Psi_{5q} &= x_{2n}^2 \left(\frac{5}{6} - \ln x_{2n} \right) - x_{1n}^2 \left(\frac{5}{6} - \ln x_{1n} \right), \;\; \Psi_{6q} = \frac{x_{2n}^{c_{3q}+1} - x_{1n}^{c_{3q}+1}}{c_{3q} + 1}, \\
q &= E \;\; \Rightarrow \;\; x_{1n} = x_{IN}, \; x_{2n} = x_E, \\
q &= M \;\; \Rightarrow \;\; x_{1n} = x_E, \; x_{2n} = x_M.
\end{aligned} \tag{4.14}$$

In case of the ellipsoidal inclusion, we get $(u_{nIN})_{x_n \to 0} \longrightarrow \pm\infty$, $(\varepsilon_{IN})_{x_n \longrightarrow 0} \longrightarrow \pm\infty$, $(\sigma_{IN})_{r \to 0} \longrightarrow \pm\infty$ due to $(\ln x_n)_{x_n \to 0} \longrightarrow \pm\infty$ and $(x_n^{c_3})_{x_n \to 0} \longrightarrow \pm\infty$ for

$c_3 < 0$ (see Equations (2.18), (4.4)–(4.10)). Accordingly, the mathematical solutions (4.4)–(4.10) are suitable for the envelope and matrix.

Additionally, the function $f = x_n\,[(1/3) - \ln x_n]$ is increasing or decreasing for $x_n \leq 0.513$ m or $x_n \geq 0.513$ m, respectively. With regard to dimensions of components of a real three-component material, $f = x_n\,[(1/3) - \ln x_n]$ represents an increasing function of x_n.

4.2 Condition $\beta_{IN} \neq \beta_E \neq \beta_M$

4.2.1 Matrix

The integration constants C_{1M}, C_{2M}, C_{3M} for the matrix (see Equation (4.4)) are determined by the boundary conditions (2.29), (2.30) or (2.29), (2.30), (2.32). The boundary conditions result in the following combinations of C_{1M}, C_{2M}, C_{3M}, i.e., $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$; $C_{1M} \neq 0$, $C_{3M} \neq 0$, $C_{2M} = 0$; $C_{2M} \neq 0$, $C_{3M} \neq 0$, $C_{1M} = 0$; $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} \neq 0$. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$. With regard to Equations (2.29), (2.30), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = \frac{p_{2n}}{\zeta}\left[\frac{2}{3} + \ln x_n + c_{3M}\left(\frac{1}{3} - \ln x_M\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{4.15}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\frac{1}{3} - \ln x_n - \left(\frac{1}{3} - \ln x_M\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{4.16}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = \left(\ln x_n - \frac{1}{3}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right) + x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M x_M^{c_{3M}-1}}\left(\frac{1}{3} - \ln x_M\right)\right], \tag{4.17}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left\{\left(\ln x_n - \frac{1}{3}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right) + x_n^{c_{3M}-1}\frac{\partial}{\partial\nu}\left[\frac{p_{2n}}{\zeta_M x_M^{c_{3M}-1}}\left(\frac{1}{3} - \ln x_M\right)\right]\right\}, \tag{4.18}$$

$$
\sigma_{nM} = \frac{p_{2n}}{\zeta_M} \left\{ \frac{2\,(c_{1M} + 2\,c_{2M})}{3} + (c_{1M} - c_{2M}) \ln x_n \right.
$$

$$
\left. + \left[(c_{1M} + c_{2M})\, c_{3M} - 2\,c_{2M}\right] \left(\frac{1}{3} - \ln x_M\right) \left(\frac{x_n}{x_M}\right)^{c_{3M}-1} \right\}, \qquad (4.19)
$$

$$
\sigma_{\varphi M} = \sigma_{\theta M} = -\,\frac{p_{2n}}{\zeta_M} \left[\frac{c_{1M} + 2\,c_{2M}}{3} - (c_{1M} - c_{2M}) \ln x_n \right.
$$

$$
\left. - (c_{1M} - c_{2M}\, c_{3M}) \left(\frac{1}{3} - \ln x_M\right) \left(\frac{x_n}{x_M}\right)^{c_{3M}-1} \right], \qquad (4.20)
$$

$$
w_M = \left(\frac{p_{2n}}{\zeta_M}\right)^2 \left\{ \kappa_{1M} + \kappa_{2M} \left(\frac{1 - 3\ln x_M}{3 x_M^{c_{3M}-1}}\right)^2 + \frac{\kappa_{4M}\,(3\ln x_M - 1)}{3 x_M^{c_{3M}-1}} \right\}
$$

$$
+ \frac{\chi_{1M}}{s_{44M}} \left\{ \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 \right\}
$$

$$
+ \frac{\chi_{2M}}{s_{44M}} \left(\left\{ \frac{\partial}{\partial\varphi} \left[\frac{p_{2n}\,(1 - 3\ln x_M)}{3\,\zeta_M\, x_M^{c_{3M}-1}} \right] \right\}^2 + \Theta^2 \left\{ \frac{\partial}{\partial\nu} \left[\frac{p_{2n}\,(1 - 3\ln x_M)}{3\,\zeta_M\, x_M^{c_{3M}-1}} \right] \right\}^2 \right)
$$

$$
+ \frac{\chi_{4M}}{s_{44M}} \; \frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right) \frac{\partial}{\partial\varphi} \left[\frac{p_{2n}\,(3\ln x_M - 1)}{3\,\zeta_M\, x_M^{c_{3M}-1}} \right]
$$

$$
+ \frac{\chi_{4M}\,\Theta^2}{s_{44M}} \; \frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right) \frac{\partial}{\partial\nu} \left[\frac{p_{2n}\,(3\ln x_M - 1)}{3\,\zeta_M\, x_M^{c_{3M}-1}} \right], \qquad (4.21)
$$

$$
W_M = 4 \int_0^{\pi/2} \int_0^{\pi/2} \left(\frac{p_{2n}}{\zeta_M}\right)^2 \left\{ \Phi_{1M} + \Phi_{2M} \left(\frac{1 - 3\ln x_M}{3 x_M^{c_{3M}-1}}\right)^2 + \frac{\Phi_{4M}\,(3\ln x_M - 1)}{3 x_M^{c_{3M}-1}} \right\} d\varphi\, d\nu
$$

$$
+ \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{1M} \left\{ \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 \right\} d\varphi\, d\nu
$$

$$
+ \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{2M} \left(\left\{ \frac{\partial}{\partial\varphi} \left[\frac{p_{2n}\,(1 - 3\ln x_M)}{3\,\zeta_M\, x_M^{c_{3M}-1}} \right] \right\}^2 \right) d\varphi\, d\nu
$$

$$
+ \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{2M}\, \Theta^2 \left(\left\{ \frac{\partial}{\partial\nu} \left[\frac{p_{2n}\,(1 - 3\ln x_M)}{3\,\zeta_M\, x_M^{c_{3M}-1}} \right] \right\}^2 \right) d\varphi\, d\nu
$$

$$+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{4M}\ \frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\left(3\ln x_M-1\right)}{3\,\zeta_M\,x_M^{c_{3M}-1}}\right]d\varphi\,d\nu$$

$$+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{4M}\,\Theta^2\ \frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left[\frac{p_{2n}\left(3\ln x_M-1\right)}{3\,\zeta_M\,x_M^{c_{3M}-1}}\right]d\varphi\,d\nu, \tag{4.22}$$

where Θ, x_M, s_{44M}, c_{iM} $(i=1,2,3)$ and κ_{jM}, χ_{jM}; Φ_{jM}, Ψ_{jM} $(j=1,2,4)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_M, ζ_{iM} $(i=1,2)$ have the forms

$$\zeta_M=\zeta_{2M}-\zeta_{1M}\left(\frac{1}{3}-\ln x_M\right),\quad \zeta_{1M}=\left[\left(c_{1M}+c_{2M}\right)c_{3M}-2\,c_{2M}\right]\left(\frac{x_E}{x_M}\right)^{c_{3M}-1},$$

$$\zeta_{2M}=-\left[\frac{2\left(c_{1M}+2\,c_{2M}\right)}{3}+\left(c_{1M}-c_{2M}\right)\ln x_E\right]. \tag{4.23}$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i,j=1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E}=-p_{2n}\,\rho_M$ and Equation (4.16), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M=\frac{1}{\zeta_M}\left[\frac{1}{3}-\ln x_E-\left(\frac{1}{3}-\ln x_M\right)\left(\frac{x_E}{x_M}\right)^{c_{3M}-1}\right]. \tag{4.24}$$

Conditions $C_{1M}\neq 0$, $C_{3M}\neq 0$, $C_{2M}=0$. With regard to Equations (2.29), (2.30), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nM}=\frac{p_{2n}}{\zeta_M\,x_M}\left(\frac{2}{3}+\ln x_n\right), \tag{4.25}$$

$$\varepsilon_{\varphi M}=\varepsilon_{\theta M}=-\frac{p_{2n}}{\zeta_M}\left[\frac{1}{x_M}\left(\frac{1}{3}-\ln x_n\right)-\frac{1}{x_n}\left(\frac{1}{3}-\ln x_M\right)\right], \tag{4.26}$$

$$\varepsilon_{n\varphi M}=s_{44M}\,\sigma_{n\varphi M}=-\left(\frac{1}{3}-\ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)+\frac{1}{x_n}\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\left(1-3\ln x_M\right)}{3\,\zeta_M}\right], \tag{4.27}$$

$$\varepsilon_{n\theta M}=s_{44M}\,\sigma_{n\theta M}=$$
$$-\Theta\left\{\left(\frac{1}{3}-\ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)-\frac{1}{x_n}\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\left(1-3\ln x_M\right)}{3\,\zeta_M}\right]\right\}, \tag{4.28}$$

$$\sigma_{nM} = \frac{p_{2n}}{\zeta_M}\left\{\frac{1}{x_M}\left[\frac{2\,(c_{1M}+2\,c_{2M})}{3}+(c_{1M}-c_{2M})\ln x_n\right]-\frac{2\,c_{2M}}{x_n}\left(\frac{1}{3}-\ln x_M\right)\right\}, \tag{4.29}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = \\ -\frac{p_{2n}}{\zeta_M}\left\{\frac{1}{x_M}\left[\frac{c_{1M}+2\,c_{2M}}{3}-(c_{1M}-c_{2M})\ln x_n\right]-\frac{c_{1M}}{x_n}\left(\frac{1}{3}-\ln x_M\right)\right\}, \tag{4.30}$$

$$\begin{aligned} w_M = {} & \left(\frac{p_{2n}}{\zeta_M}\right)^2\left[\frac{\kappa_{1M}}{x_M^2}+\kappa_{3M}\left(\frac{1}{3}-\ln x_M\right)^2+\frac{\kappa_{5M}\,(3\ln x_M-1)}{3x_M}\right] \\ & +\frac{\chi_{1M}}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)\right]^2\right\} \\ & +\frac{\chi_{3M}}{s_{44M}}\left(\left\{\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\,(1-3\ln x_M)}{3\,\zeta_M}\right]\right\}^2+\Theta^2\left\{\frac{\partial}{\partial\nu}\left[\frac{p_{2n}\,(1-3\ln x_M)}{3\,\zeta_M}\right]\right\}^2\right) \\ & +\frac{\chi_{5M}}{s_{44M}}\,\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\,(3\ln x_M-1)}{3\,\zeta_M}\right] \\ & +\frac{\chi_{5M}\,\Theta^2}{s_{44M}}\,\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)\frac{\partial}{\partial\nu}\left[\frac{p_{2n}\,(3\ln x_M-1)}{3\,\zeta_M}\right], \end{aligned} \tag{4.31}$$

$$\begin{aligned} W_M = {} & 4\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\left(\frac{p_{2n}}{\zeta_M}\right)^2\left[\frac{\Phi_{1M}}{x_M^2}+\Phi_{3M}\left(\frac{1}{3}-\ln x_M\right)^2+\frac{\Phi_{5M}\,(3\ln x_M-1)}{3x_M}\right]d\varphi\,d\nu \\ & +\frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\Psi_{1M}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)\right]^2\right\}d\varphi\,d\nu \\ & +\frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\Psi_{3M}\left\{\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\,(1-3\ln x_M)}{3\,\zeta_M}\right]\right\}^2 d\varphi\,d\nu \\ & +\frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\Psi_{3M}\,\Theta^2\left\{\frac{\partial}{\partial\nu}\left[\frac{p_{2n}\,(1-3\ln x_M)}{3\,\zeta_M}\right]\right\}^2 d\varphi\,d\nu \\ & +\frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\Psi_{5M}\,\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\,(3\ln x_M-1)}{3\,\zeta_M}\right]d\varphi\,d\nu \end{aligned}$$

$$+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{5M}\,\Theta^2\,\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\,x_M}\right)\frac{\partial}{\partial\nu}\left[\frac{p_{2n}\,(3\ln x_M-1)}{3\,\zeta_M}\right]\,d\varphi\,d\nu,\tag{4.32}$$

where Θ, x_M, s_{44M}, c_{iM} $(i=1,2,3)$ and κ_{jM}, χ_{jM}; Φ_{jM}, Ψ_{jM} $(j=1,3,5)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_M have the forms

$$\zeta_M=\frac{1}{x_M}\left[\frac{2\,(c_{1M}+2\,c_{2M})}{3}+(c_{1M}-c_{2M})\ln x_E\right]+\frac{2\,c_{2M}}{x_E}\left(\frac{1}{3}-\ln x_M\right).\tag{4.33}$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i,j=1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E}=-p_{2n}\,\rho_M$ and Equation (4.26), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M=\frac{1}{\zeta_M}\left[\frac{1}{x_M}\left(\frac{1}{3}-\ln x_E\right)-\frac{1}{x_E}\left(\frac{1}{3}-\ln x_M\right)\right].\tag{4.34}$$

Conditions $C_{2M}\neq 0$, $C_{3M}\neq 0$, $C_{1M}=0$. With regard to Equations (2.29), (2.30), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nM}=-\frac{p_{2n}\,c_{3M}}{\zeta_M\,x_M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1},\tag{4.35}$$

$$\varepsilon_{\varphi M}=\varepsilon_{\theta M}=\frac{p_{2n}}{\zeta_M\,x_n}\left[1-\left(\frac{x_n}{x_M}\right)^{c_{3M}}\right],\tag{4.36}$$

$$\varepsilon_{n\varphi M}=s_{44M}\,\sigma_{n\varphi M}=-\frac{1}{x_n}\left[x_n^{c_{3M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\,x_M^{c_{3M}}}\right)-\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right],\tag{4.37}$$

$$\varepsilon_{n\theta M}=s_{44M}\,\sigma_{n\theta M}=-\frac{\Theta}{x_n}\left[x_n^{c_{3M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\,x_M^{c_{3M}}}\right)-\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right],\tag{4.38}$$

$$\sigma_{nM}=-\frac{p_{2n}}{\zeta_M}\left\{\frac{c_{3M}\,(c_{1M}+c_{2M})-2\,c_{2M}}{x_M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}+\frac{2\,c_{2M}}{x_n}\right\},\tag{4.39}$$

$$\sigma_{\varphi M}=\sigma_{\theta M}=-\frac{p_{2n}}{\zeta_M}\left[\frac{c_{1M}-c_{2M}\,c_{3M}}{x_M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}-\frac{c_{1M}}{x_n}\right],\tag{4.40}$$

$$
\begin{aligned}
w_M = {} & \left(\frac{p_{2n}}{\zeta_M}\right)^2 \left(\frac{\kappa_{2M}}{x_M^{2c_{3M}}} + \kappa_{3M} - \frac{\kappa_{6M}}{x_M^{c_{3M}}}\right) \\
& + \frac{\chi_{2M}}{s_{44M}} \left\{ \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}}}\right)\right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}}}\right)\right]^2 \right\} \\
& + \frac{\chi_{3M}}{s_{44M}} \left\{ \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 \right\} \\
& - \frac{\chi_{6M}}{s_{44M}} \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}}}\right) \frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right) + \Theta^2 \frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}}}\right) \frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right],
\end{aligned}
\tag{4.41}
$$

$$
\begin{aligned}
W_M = {} & 4 \int_0^{\pi/2}\int_0^{\pi/2} \left(\frac{p_{2n}}{\zeta_M}\right)^2 \left(\frac{\Phi_{2M}}{x_M^{2c_{3M}}} + \Phi_{3M} - \frac{\Phi_{6M}}{x_M^{c_{3M}}}\right) d\varphi\, d\nu \\
& + \frac{4}{s_{44M}} \int_0^{\pi/2}\int_0^{\pi/2} \Psi_{2M} \left\{ \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}}}\right)\right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}}}\right)\right]^2 \right\} d\varphi\, d\nu \\
& + \frac{4}{s_{44M}} \int_0^{\pi/2}\int_0^{\pi/2} \Psi_{3M} \left\{ \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 \right\} d\varphi\, d\nu \\
& - \frac{4}{s_{44M}} \int_0^{\pi/2}\int_0^{\pi/2} \Psi_{6M} \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}}}\right) \frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right. \\
& \qquad \left. + \Theta^2 \frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}}}\right) \frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right] d\varphi\, d\nu,
\end{aligned}
\tag{4.42}
$$

where Θ, x_M, s_{44M}, c_{iM} $(i=1,2,3)$ and κ_{jM}, χ_{jM}; Φ_{jM}, Ψ_{jM} $(j=2,3,6)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_M has the form

$$
\zeta_M = \frac{1}{x_E}\left\{\left[c_{3M}\left(c_{1M}+c_{2M}\right) - 2\,c_{2M}\right]\left(\frac{x_E}{x_M}\right)^{c_{3M}} + 2\,c_{2M}\right\}. \tag{4.43}
$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i,j=1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E} = -p_{2n}\,\rho_M$ and Equation (4.36), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M = \frac{1}{\zeta_M}\left[\left(\frac{x_E}{x_M}\right)^{c_{3M}} - 1\right]. \tag{4.44}$$

Conditions $C_{1M} \neq 0$**,** $C_{2M} \neq 0$**,** $C_{3M} \neq 0$**.** With regard to Equations (2.29), (2.30), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M}\left[\zeta_{1M}\left(\frac{2}{3} + \ln x_n\right) - \zeta_{2M}\, c_{3M}\, x_n^{c_{3M}-1}\right], \tag{4.45}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\zeta_{1M}\left(\frac{1}{3} - \ln x_n\right) + \zeta_{2M}\, x_n^{c_{3M}-1} + \frac{\zeta_{3M}}{x_n}\right], \tag{4.46}$$

$$\begin{aligned} \varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = \left(\frac{1}{3} - \ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right) + x_n^{c_3-1}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right) \\ + \frac{1}{x_n}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right), \end{aligned} \tag{4.47}$$

$$\begin{aligned} \varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left[\left(\frac{1}{3} - \ln x_n\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right) + x_n^{c_3-1}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right) \right.\\ \left. + \frac{1}{x_n}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right], \end{aligned} \tag{4.48}$$

$$\begin{aligned} \sigma_{nM} = -\frac{p_{2n}}{\zeta_M}\left\{\zeta_{1M}\left[\frac{2\,(c_{1M} + 2\,c_{2M})}{3} + (c_{1M} - c_{2M})\ln x_n\right]\right.\\ \left. - \zeta_{2M}\left[(c_{1M} + c_{2M})\,c_{3M} - 2\,c_{2M}\right]x_n^{c_{3M}-1} + \frac{2\,c_{2M}\,\zeta_{3M}}{x_n}\right\}, \end{aligned} \tag{4.49}$$

$$\begin{aligned} \sigma_{\varphi M} = \sigma_{\theta M} = \frac{p_{2n}}{\zeta_M}\left\{\zeta_{1M}\left[\frac{c_{1M} + 2\,c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right]\right.\\ \left. + \zeta_{2M}\,(c_{1M} - c_{2M}\,c_{3M})\,x_n^{c_{3M}-1} + \frac{c_{1M}\,\zeta_{3M}}{x_n}\right\}, \end{aligned} \tag{4.50}$$

$$\begin{aligned} w_M = \left(\frac{p_{2n}}{\zeta_M}\right)^2 \left(\kappa_{1M}\,\zeta_{1M}^2 + \kappa_{2M}\,\zeta_{2M}^2 + \kappa_{3M}\,\zeta_{3M}^2\right.\\ \left. + \kappa_{4M}\,\zeta_{1M}\,\zeta_{2M} + \kappa_{5M}\,\zeta_{1M}\,\zeta_{3M} + \kappa_{6M}\,\zeta_{2M}\,\zeta_{3M}\right) \end{aligned}$$

$$
\begin{aligned}
&+\frac{\chi_{1M}}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\right]^2\right\}\\
&+\frac{\chi_{2M}}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\right]^2\right\}\\
&+\frac{\chi_{3M}}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right]^2\right\}\\
&+\frac{\chi_{4M}}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\\
&+\frac{\chi_{4M}\,\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\\
&+\frac{\chi_{5M}}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\\
&+\frac{\chi_{5M}\,\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\\
&+\frac{\chi_{6M}}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\\
&+\frac{\chi_{6M}\,\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right),
\end{aligned}
\tag{4.51}
$$

$$
\begin{aligned}
W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\frac{p_{2n}}{\zeta_M}\right)^2\Big(&\Phi_{1M}\zeta_{1M}^2+\Phi_{2M}\zeta_{2M}^2+\Phi_{3M}\zeta_{3M}^2+\Phi_{4M}\zeta_{1M}\zeta_{2M}\\
&+\Phi_{5M}\zeta_{1M}\zeta_{3M}+\Phi_{6M}\zeta_{2M}\zeta_{3M}\Big)\,d\varphi\,d\nu\\
+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1M}&\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\right]^2\right\}d\varphi\,d\nu\\
+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2M}&\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\right]^2\right\}d\varphi\,d\nu\\
+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{3M}&\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right]^2\right\}d\varphi\,d\nu\\
+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{4M}&\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\right.
\end{aligned}
$$

$$+ \Theta^2 \frac{\partial}{\partial \nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial \nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\Bigg] d\varphi\, d\nu$$

$$+ \frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} \Psi_{5M}\left[\frac{\partial}{\partial \varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial \varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right.$$

$$\left. + \Theta^2 \frac{\partial}{\partial \nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial \nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right] d\varphi\, d\nu$$

$$+ \frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} \Psi_{6M}\left[\frac{\partial}{\partial \varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\frac{\partial}{\partial \varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right.$$

$$\left. + \Theta^2 \frac{\partial}{\partial \nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\frac{\partial}{\partial \nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right] d\varphi\, d\nu, \qquad (4.52)$$

where Θ, x_M, s_{44M}, c_{iM} $(i=1,2,3)$ and κ_{iM}, χ_{iM}; Φ_{iM}, Ψ_{iM} $(i=1,\ldots,6)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_{iM} $(i=1,2,3)$, ζ_M have the forms

$$\zeta_{1M} = c_{3M} x_M^{c_{3M}-1}, \quad \zeta_{2M} = \frac{2}{3} + \ln x_M,$$

$$\zeta_{3M} = -x_M^{c_{3M}}\left[\frac{2}{3} + \ln x_M + c_{3M}\left(\frac{1}{3} - \ln x_M\right)\right],$$

$$\zeta_M = c_{3M}\left\{\left[\frac{2\left(c_{1M}+2c_{2M}\right)}{3} + \left(c_{1M}-c_{2M}\right)\ln x_E\right] - \frac{2c_{2M}x_M}{x_E}\left(\frac{1}{3} - \ln x_M\right)\right\} x_M^{c_{3M}-1}$$

$$- \left\{\left[\left(c_{1M}+c_{2M}\right)c_{3M} - 2c_{2M}\right] x_E^{c_{3M}-1} + \frac{2c_{2M}x_M^{c_{3M}}}{x_E}\right\}\left(\frac{2}{3} + \ln x_M\right). \qquad (4.53)$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i,j=1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E} = -p_{2n}\rho_M$ and Equation (4.46), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M = \frac{1}{\zeta_M}\left[\zeta_{1M}\left(\ln x_E - \frac{1}{3}\right) - \zeta_{2M} x_E^{c_{3M}-1} - \frac{\zeta_{3M}}{x_E}\right]. \qquad (4.54)$$

4.2.2 Envelope

The integration constants C_{1E}, C_{2E}, C_{3E} for the envelope (see Equation (4.4)) are determined by the boundary conditions (2.33), (2.34). The boundary conditions result in the following combinations of C_{1E}, C_{2E}, C_{3E}, i.e., $C_{1E} \neq 0$, $C_{2E} \neq 0$, $C_{3E} = 0$; $C_{1E} \neq 0$, $C_{3E} \neq 0$, $C_{2E} = 0$; $C_{2E} \neq 0$, $C_{3E} \neq 0$, $C_{1E} = 0$. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1E} \neq 0$, $C_{2E} \neq 0$, $C_{3E} = 0$. With regard to Equations (2.33), (2.34), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = \zeta_{1E}\left(\frac{2}{3} + \ln x_n\right) - c_{3E}\,\zeta_{2E}\,x_n^{c_{3E}-1}, \tag{4.55}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\left[\zeta_{1E}\left(\frac{1}{3} - \ln x_n\right) + \zeta_{2E}\,x_n^{c_{3E}-1}\right], \tag{4.56}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\left[\frac{\partial \zeta_{1E}}{\partial \varphi}\left(\frac{1}{3} - \ln x_n\right) + \frac{\partial \zeta_{2E}}{\partial \varphi}\,x_n^{c_{3E}-1}\right], \tag{4.57}$$

$$\varepsilon_{n\theta E} = -\Theta\left[\frac{\partial \zeta_{1E}}{\partial \nu}\left(\frac{1}{3} - \ln x_n\right) + \frac{\partial \zeta_{2E}}{\partial \nu}\,x_n^{c_{3E}-1}\right], \tag{4.58}$$

$$\begin{aligned}\sigma_{nE} = -\Bigg\{\zeta_{1E}&\left[\frac{2\left(c_{1E} + 2c_{2E}\right)}{3} + \left(c_{1E} - c_{2E}\right)\ln x_n\right] \\ &+ \zeta_{2E}\left[\left(c_{1E} + c_{2E}\right)c_{3E} - 2c_{2E}\right]x_n^{c_{3E}-1}\Bigg\},\end{aligned} \tag{4.59}$$

$$\begin{aligned}\sigma_{\varphi E} = \sigma_{\theta E} = -\Bigg\{\zeta_{1E}&\left[\frac{c_{1E} + 2c_{2E}}{3} - \left(c_{1E} - c_{2E}\right)\ln x_n\right] \\ &+ \zeta_{2E}\left(c_{1E} - c_{2E}\,c_{3E}\right)x_n^{c_{3E}-1}\Bigg\},\end{aligned} \tag{4.60}$$

$$\begin{aligned}w_E = {}& \kappa_{1E}\,\zeta_{1E}^2 + \kappa_{2E}\,\zeta_{2E}^2 + \kappa_{4E}\,\zeta_{1E}\,\zeta_{2E} + \frac{\chi_{1E}}{s_{44E}}\left[\left(\frac{\partial \zeta_{1E}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{1E}}{\partial \nu}\right)^2\right] \\ &+ \frac{\chi_{2E}}{s_{44E}}\left[\left(\frac{\partial \zeta_{2E}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{2E}}{\partial \nu}\right)^2\right] + \frac{\chi_{4E}}{s_{44E}}\left(\frac{\partial \zeta_{1E}}{\partial \varphi}\frac{\partial \zeta_{2E}}{\partial \varphi} + \Theta^2\frac{\partial \zeta_{1E}}{\partial \nu}\frac{\partial \zeta_{2E}}{\partial \nu}\right),\end{aligned} \tag{4.61}$$

$$
\begin{aligned}
W_E = {} & 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{1E}\,\zeta_{1E}^2+\Phi_{2E}\,\zeta_{2E}^2+\Phi_{4E}\,\zeta_{1E}\,\zeta_{2E}\right)d\varphi\,d\nu \\
& + \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1E}\left[\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{1E}}{\partial\nu}\right)^2\right]d\varphi\,d\nu \\
& + \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2E}\left[\left(\frac{\partial\zeta_{2E}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{2E}}{\partial\nu}\right)^2\right]d\varphi\,d\nu \\
& + \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{4E}\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\frac{\partial\zeta_{2E}}{\partial\varphi}+\Theta^2\frac{\partial\zeta_{1E}}{\partial\nu}\frac{\partial\zeta_{2E}}{\partial\nu}\right)d\varphi\,d\nu, \qquad (4.62)
\end{aligned}
$$

where Θ, x_E, s_{44E}, c_{iE} ($i=1,2,3$) and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} ($j=1,2,4$) are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_{kE} ($k=1,2$) has the form

$$
\begin{aligned}
\zeta_{iE} &= p_{1n}\,\zeta_{i1E}+p_{2n}\,\zeta_{i2E}, \quad i=1,2, \\
\zeta_{11E} &= \frac{1}{\zeta_E}\left[(c_{1E}+c_{2E})\,c_{3E}-2\,c_{2E}\right]x_E^{c_{3E}-1}, \\
\zeta_{12E} &= -\frac{1}{\zeta_E}\left[(c_{1E}+c_{2E})\,c_{3E}-2\,c_{2E}\right]x_{IN}^{c_{3E}-1}, \\
\zeta_{21E} &= \frac{1}{\zeta_E}\left[\frac{2\,(c_{1E}+2\,c_{2E})}{3}+(c_{1E}-c_{2E})\ln x_E\right], \\
\zeta_{22E} &= -\frac{1}{\zeta_E}\left[\frac{2\,(c_{1E}+2\,c_{2E})}{3}+(c_{1E}-c_{2E})\ln x_{IN}\right], \\
\zeta_E &= \left[(c_{1E}+c_{2E})\,c_{3E}-2\,c_{2E}\right] \\
& \quad \times\left[\frac{2\,(c_{1E}+2\,c_{2E})}{3}\left(x_{IN}^{c_{3E}-1}-x_E^{c_{3E}-1}\right)\right. \\
& \qquad \left. +\ (c_{1E}-c_{2E})\left(x_{IN}^{c_{3E}-1}\ln x_E-x_E^{c_{3E}-1}\ln x_{IN}\right)\right]. \qquad (4.63)
\end{aligned}
$$

The normal stresses p_{1n}, p_{2n} in Equation (4.23) are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (4.24), (4.34), (4.44), (4.54), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}} = -\left(p_{1n}\,\rho_{11E}+p_{2n}\,\rho_{12E}\right)$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E} = -\left(p_{1n}\,\rho_{21E}+p_{2n}\,\rho_{22E}\right)$ and Equation (4.56), the coefficient ρ_{ijE} ($i,j=1,2$) in Equations (2.25), (2.26) is derived as

$$\rho_{1iE} = \zeta_{1iE}\left(\frac{1}{3} - \ln x_{IN}\right) + \zeta_{2iE}\, x_{IN}^{c_{3E}-1}, \quad \rho_{2iE} = \zeta_{1iE}\left(\frac{1}{3} - \ln x_E\right) + \zeta_{2iE}\, x_E^{c_{3E}-1},$$
$$i = 1, 2. \tag{4.64}$$

Conditions $C_{1E} \neq 0$, $C_{3E} \neq 0$, $C_{2E} = 0$. With regard to Equations (2.33), (2.34), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = \zeta_{1E}\left(\frac{2}{3} + \ln x_n\right), \tag{4.65}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\left[\zeta_{1E}\left(\frac{1}{3} - \ln x_n\right) + \frac{\zeta_{2E}}{x_n}\right], \tag{4.66}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\left[\frac{\partial \zeta_{1E}}{\partial \varphi}\left(\frac{1}{3} - \ln x_n\right) + \frac{1}{x_n}\frac{\partial \zeta_{2E}}{\partial \varphi}\right], \tag{4.67}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\Theta\left[\frac{\partial \zeta_{1E}}{\partial \nu}\left(\frac{1}{3} - \ln x_n\right) + \frac{1}{x_n}\frac{\partial \zeta_{2E}}{\partial \nu}\right], \tag{4.68}$$

$$\sigma_{nE} = -\left\{\zeta_{1E}\left[\frac{2\left(c_{1E} + 2c_{2E}\right)}{3} + \left(c_{1E} - c_{2E}\right)\ln x_n\right] - \frac{2c_{2E}\,\zeta_{2E}}{x_n}\right\}, \tag{4.69}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\left\{\zeta_{1E}\left[\frac{c_{1E} + 2c_{2E}}{3} - \left(c_{1E} - c_{2E}\right)\ln x_n\right] + \frac{c_{1E}\,\zeta_{2E}}{x_n}\right\}, \tag{4.70}$$

$$w_E = \kappa_{1E}\,\zeta_{1E}^2 + \kappa_{3E}\,\zeta_{3E}^2 + \kappa_{5E}\,\zeta_{1E}\,\zeta_{3E} + \frac{\chi_{1E}}{s_{44E}}\left[\left(\frac{\partial \zeta_{1E}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{1E}}{\partial \nu}\right)^2\right]$$
$$+ \frac{\chi_{3E}}{s_{44E}}\left[\left(\frac{\partial \zeta_{3E}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{3E}}{\partial \nu}\right)^2\right] + \frac{\chi_{5E}}{s_{44E}}\left(\frac{\partial \zeta_{1E}}{\partial \varphi}\frac{\partial \zeta_{3E}}{\partial \varphi} + \Theta^2\frac{\partial \zeta_{1E}}{\partial \nu}\frac{\partial \zeta_{3E}}{\partial \nu}\right), \tag{4.71}$$

$$
\begin{aligned}
W_E = {} & 4 \int_0^{\pi/2} \int_0^{\pi/2} \left(\Phi_{1E}\, \zeta_{1E}^2 + \Phi_{3E}\, \zeta_{3E}^2 + \Phi_{5E}\, \zeta_{1E}\, \zeta_{3E} \right) d\varphi\, d\nu \\
& + \frac{4}{s_{44E}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{1E} \left[\left(\frac{\partial \zeta_{1E}}{\partial \varphi} \right)^2 + \Theta^2 \left(\frac{\partial \zeta_{1E}}{\partial \nu} \right)^2 \right] d\varphi\, d\nu \\
& + \frac{4}{s_{44E}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{3E} \left[\left(\frac{\partial \zeta_{3E}}{\partial \varphi} \right)^2 + \Theta^2 \left(\frac{\partial \zeta_{3E}}{\partial \nu} \right)^2 \right] d\varphi\, d\nu \\
& + \frac{4}{s_{44E}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{5E} \left(\frac{\partial \zeta_{1E}}{\partial \varphi} \frac{\partial \zeta_{3E}}{\partial \varphi} + \Theta^2 \frac{\partial \zeta_{1E}}{\partial \nu} \frac{\partial \zeta_{3E}}{\partial \nu} \right) d\varphi\, d\nu, \qquad (4.72)
\end{aligned}
$$

where Θ, x_E, s_{44E}, c_{iE} $(i=1,2,3)$ and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} $(j=1,3,5)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_{kE} $(k=1,3)$ has the form

$$
\begin{aligned}
& \zeta_{iE} = p_{1n}\, \zeta_{i1E} + p_{2n}\, \zeta_{i2E}, \quad i = 1, 3, \\
& \zeta_{11E} = -\frac{2\, c_{2E}}{\zeta_E\, x_E}, \quad \zeta_{12E} = \frac{2\, c_{2E}}{\zeta_E\, x_{IN}}, \\
& \zeta_{31E} = \frac{1}{\zeta_E} \left[\frac{2\,(c_{1E} + 2\, c_{2E})}{3} + (c_{1E} - c_{2E}) \ln x_E \right], \\
& \zeta_{32E} = -\frac{1}{\zeta_E} \left[\frac{2\,(c_{1E} + 2\, c_{2E})}{3} + (c_{1E} - c_{2E}) \ln x_{IN} \right], \\
& \zeta_E = 2\, c_{2E} \left[\frac{2\,(c_{1E} + 2\, c_{2E})}{3} \left(\frac{1}{x_E} - \frac{1}{x_{IN}} \right) + (c_{1E} - c_{2E}) \left(\frac{\ln x_{IN}}{x_E} - \frac{\ln x_E}{x_{IN}} \right) \right]. \qquad (4.73)
\end{aligned}
$$

The normal stresses p_{1n}, p_{2n} in Equation (4.33) are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (4.24), (4.34), (4.44), (4.54), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n = x_{IN}} = -\left(p_{1n}\, \rho_{11E} + p_{2n}\, \rho_{12E}\right)$, $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E} = -\left(p_{1n}\, \rho_{21E} + p_{2n}\, \rho_{22E}\right)$ and Equation (4.66), the coefficient ρ_{ijE} $(i, j = 1,2)$ in Equations (2.25), (2.26) is derived as

$$
\begin{aligned}
& \rho_{1iE} = \zeta_{1iE} \left(\frac{1}{3} - \ln x_{IN} \right) + \frac{\zeta_{3iE}}{x_{IN}}, \quad \rho_{2iE} = \zeta_{1iE} \left(\frac{1}{3} - \ln x_E \right) + \frac{\zeta_{3iE}}{x_E}, \\
& i = 1, 2. \qquad (4.74)
\end{aligned}
$$

Conditions $C_{2E} \neq 0$**,** $C_{3E} \neq 0$**,** $C_{1E} = 0$**.** With regard to Equations (2.33), (2.34), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = -c_{3E}\,\zeta_{2E}\,x_n^{c_{3E}-1}, \tag{4.75}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\left(\zeta_{2E}\,x_n^{c_{3E}-1} + \frac{\zeta_{3E}}{x_n}\right), \tag{4.76}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\left(\frac{\partial \zeta_{2E}}{\partial \varphi}\,x_n^{c_{3E}-1} + \frac{1}{x_n}\,\frac{\partial \zeta_{3E}}{\partial \varphi}\right), \tag{4.77}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\Theta\left(\frac{\partial \zeta_{2E}}{\partial \nu}\,x_n^{c_{3E}-1} + \frac{1}{x_n}\,\frac{\partial \zeta_{3E}}{\partial \nu}\right), \tag{4.78}$$

$$\sigma_{nE} = -\left[\zeta_{2E}\,\left[(c_{1E}+c_{2E})\,c_{3E} - 2\,c_{2E}\right]x_n^{c_{3E}-1} - \frac{2\,c_{2E}\,\zeta_{3E}}{x_n}\right], \tag{4.79}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\left[\zeta_{2E}\,\left(c_{1E} - c_{2E}\,c_{3E}\right)x_n^{c_{3E}-1} + \frac{c_{1E}\,\zeta_{3E}}{x_n}\right], \tag{4.80}$$

$$\begin{aligned} w_E &= \kappa_{2E}\,\zeta_{2E}^2 + \kappa_{3E}\,\zeta_{3E}^2 + \kappa_{6E}\,\zeta_{2E}\,\zeta_{3E} + \frac{\chi_{2E}}{s_{44E}}\left[\left(\frac{\partial \zeta_{2E}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{2E}}{\partial \nu}\right)^2\right] \\ &+ \frac{\chi_{3E}}{s_{44E}}\left[\left(\frac{\partial \zeta_{3E}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{3E}}{\partial \nu}\right)^2\right] + \frac{\chi_{6E}}{s_{44E}}\left(\frac{\partial \zeta_{2E}}{\partial \varphi}\,\frac{\partial \zeta_{3E}}{\partial \varphi} + \Theta^2\frac{\partial \zeta_{2E}}{\partial \nu}\,\frac{\partial \zeta_{3E}}{\partial \nu}\right), \end{aligned} \tag{4.81}$$

$$\begin{aligned} W_E &= 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{2E}\,\zeta_{2E}^2 + \Phi_{3E}\,\zeta_{3E}^2 + \Phi_{6E}\,\zeta_{2E}\,\zeta_{3E}\right)d\varphi\,d\nu \\ &+ \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2E}\left[\left(\frac{\partial \zeta_{2E}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{2E}}{\partial \nu}\right)^2\right]d\varphi\,d\nu \\ &+ \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{3E}\left[\left(\frac{\partial \zeta_{3E}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{3E}}{\partial \nu}\right)^2\right]d\varphi\,d\nu \\ &+ \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{6E}\left(\frac{\partial \zeta_{2E}}{\partial \varphi}\,\frac{\partial \zeta_{3E}}{\partial \varphi} + \Theta^2\frac{\partial \zeta_{2E}}{\partial \nu}\,\frac{\partial \zeta_{3E}}{\partial \nu}\right)d\varphi\,d\nu, \end{aligned} \tag{4.82}$$

where Θ, x_E, s_{44E}, c_{iE} $(i=1,2,3)$ and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} $(j=2,3,6)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_{kE} $(k=1,2)$ has the form

$$\begin{aligned}
&\zeta_{iE} = p_{1n}\zeta_{i1E} + p_{2n}\zeta_{i2E}, \quad i=2,3,\\
&\zeta_{21E} = -\frac{2c_{2E}}{\zeta_E x_E}, \quad \zeta_{22E} = \frac{2c_{2E}}{\zeta_E x_{IN}},\\
&\zeta_{31E} = -\frac{[(c_{1E}+c_{2E})c_{3E} - 2c_{2E}]x_E^{c_{3E}-1}}{\zeta_E},\\
&\zeta_{32E} = \frac{[(c_{1E}+c_{2E})c_{3E} - 2c_{2E}]x_{IN}^{c_{3E}-1}}{\zeta_E},\\
&\zeta_E = 2c_{2E}\left[(c_{1E}+c_{2E})c_{3E} - 2c_{2E}\right]\left(\frac{x_E^{c_{3E}-1}}{x_{IN}} - \frac{x_{IN}^{c_{3E}-1}}{x_E}\right).
\end{aligned} \tag{4.83}$$

The normal stresses p_{1n}, p_{2n} in Equation (4.33) are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (4.24), (4.34), (4.44), (4.54), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}} = -\left(p_{1n}\rho_{11E} + p_{2n}\rho_{12E}\right)$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E} = -\left(p_{1n}\rho_{21E} + p_{2n}\rho_{22E}\right)$ and Equation (4.76), the coefficient ρ_{ijE} $(i,j=1,2)$ in Equations (2.25), (2.26) is derived as

$$\rho_{1iE} = \zeta_{2iE}x_{IN}^{c_{3E}-1} + \frac{\zeta_{3iE}}{x_{IN}}, \quad \rho_{2iE} = \zeta_{2iE}x_E^{c_{3E}-1} + \frac{\zeta_{3iE}}{x_E}, \quad i=1,2. \tag{4.84}$$

4.3 Condition $\beta_{IN} \neq \beta_E = \beta_M$

4.3.1 Matrix

The integration constants C_{1M}, C_{2M}, C_{3M} for the matrix (see Equation (4.4)) are determined by the boundary conditions (2.30), (2.31) or (2.30)–(2.32). The boundary conditions result in the following combinations of C_{1M}, C_{2M}, C_{3M}, i.e., $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$; $C_{1M} \neq 0$, $C_{3M} \neq 0$, $C_{2M} = 0$; $C_{2M} \neq 0$, $C_{3M} \neq 0$, $C_{1M} = 0$; $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} \neq 0$. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$. With regard to Equations (2.30), (2.31), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\zeta_{1M}\left(\frac{2}{3} + \ln x_n\right) + c_{3M}\,\zeta_{2M}\,x_n^{c_{3M}-1}, \tag{4.85}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = \zeta_{1M}\left(\frac{1}{3} - \ln x_n\right) + \zeta_{2M}\,x_n^{c_{3M}-1}, \tag{4.86}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = \left(\frac{1}{3} - \ln x_n\right)\frac{\partial\zeta_{1M}}{\partial\varphi} + x_n^{c_{3M}-1}\frac{\partial\zeta_{2M}}{\partial\varphi}, \tag{4.87}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = \Theta\left[\left(\frac{1}{3} - \ln x_n\right)\frac{\partial\zeta_{1M}}{\partial\nu} + x_n^{c_{3M}-1}\frac{\partial\zeta_{2M}}{\partial\nu},\right], \tag{4.88}$$

$$\begin{aligned}\sigma_{nM} = &-\zeta_{1M}\left[\frac{2\,(c_{1M} + 2\,c_{2M})}{3} + (c_{1M} - c_{2M})\ln x_n\right]\\ &+\zeta_{2M}\left[(c_{1M} + c_{2M})\,c_{3M} - 2\,c_{2M}\right]x_n^{c_{3M}-1},\end{aligned} \tag{4.89}$$

$$\begin{aligned}\sigma_{\varphi M} = \sigma_{\theta M} = \zeta_{1M}&\left[\frac{c_{1M} + 2\,c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right]\\ &+\zeta_{2M}\,(c_{1M} - c_{2M}\,c_{3M})\,x_n^{c_{3M}-1},\end{aligned} \tag{4.90}$$

$$\begin{aligned}w_M = {}&\kappa_{1M}\,\zeta_{1M}^2 + \kappa_{2M}\,\zeta_{2M}^2 + \kappa_{4M}\,\zeta_{1M}\,\zeta_{2M} + \frac{\chi_{1M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{1M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{1M}}{\partial\nu}\right)^2\right]\\ &+\frac{\chi_{2M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{2M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2M}}{\partial\nu}\right)^2\right] + \frac{\chi_{4M}}{s_{44M}}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\,\frac{\partial\zeta_{2M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\,\frac{\partial\zeta_{2M}}{\partial\nu}\right],\end{aligned} \tag{4.91}$$

$$\begin{aligned}W_M = {}&4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{1M}\,\zeta_{1M}^2 + \Phi_{2M}\,\zeta_{2M}^2 + \Phi_{4M}\,\zeta_{1M}\,\zeta_{2M}\right)d\varphi\,d\nu\\ &+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1M}\left[\left(\frac{\partial\zeta_{1M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{1M}}{\partial\nu}\right)^2\right]d\varphi\,d\nu\\ &+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2M}\left[\left(\frac{\partial\zeta_{2M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2M}}{\partial\nu}\right)^2\right]d\varphi\,d\nu\\ &+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{4M}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\,\frac{\partial\zeta_{2M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\,\frac{\partial\zeta_{2M}}{\partial\nu}\right]d\varphi\,d\nu,\end{aligned} \tag{4.92}$$

where Θ, x_E, x_M, s_{44M}, c_{iM} and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} ($i=1,2,3;\ j=1,2,4$) are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_{1M}, ζ_{2M}, ζ_M have the forms

$$\zeta_{1M} = -\frac{p_{1n}\,\rho_{2ME}\,x_E\,x_M^{c_{3M}}}{\zeta_M}, \quad \zeta_{2M} = \frac{p_{1n}\,\rho_{2ME}\,x_E\,x_M}{\zeta_M}\left(\frac{1}{3} - \ln x_M\right),$$
$$\zeta_M = x_M^{c_{3M}}\,x_E\left(\frac{1}{3} - \ln x_E\right) - x_M\,x_E^{c_{3M}}\left(\frac{1}{3} - \ln x_M\right). \tag{4.93}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{1ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{3M} \neq 0$, $C_{2M} = 0$. With regard to Equations (2.30), (2.31), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\,\zeta_{1M}\left(\frac{2}{3} + \ln x_n\right), \tag{4.94}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = \zeta_{1M}\left(\frac{1}{3} - \ln x_n\right) + \frac{\zeta_{3M}}{x_n}, \tag{4.95}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = \left(\frac{1}{3} - \ln x_n\right)\frac{\partial\zeta_{1M}}{\partial\varphi} + \frac{1}{x_n}\frac{\partial\zeta_{3M}}{\partial\varphi}, \tag{4.96}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = \Theta\left[\left(\frac{1}{3} - \ln x_n\right)\frac{\partial\zeta_{1M}}{\partial\nu} + \frac{1}{x_n}\frac{\partial\zeta_{3M}}{\partial\nu}\right], \tag{4.97}$$

$$\sigma_{nM} = -\,\zeta_{1M}\left[\frac{2\,(c_{1M} + 2\,c_{2M})}{3} + (c_{1M} - c_{2M})\ln x_n\right] - \frac{\zeta_{3M}}{x_n}, \tag{4.98}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = \zeta_{1M}\left[\frac{c_{1M} + 2\,c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right] + \frac{c_{1M}\,\zeta_{3M}}{x_n}, \tag{4.99}$$

$$w_M = \kappa_{1M}\,\zeta_{1M}^2 + \kappa_{3M}\,\zeta_{3M}^2 + \kappa_{5M}\,\zeta_{1M}\,\zeta_{3M} + \frac{\chi_{1M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{1M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{1M}}{\partial\nu}\right)^2\right]$$
$$+ \frac{\chi_{3M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{3M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{3M}}{\partial\nu}\right)^2\right] + \frac{\chi_{5M}}{s_{44M}}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\frac{\partial\zeta_{3M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\frac{\partial\zeta_{3M}}{\partial\nu}\right], \tag{4.100}$$

$$
\begin{aligned}
W_M = {} & 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{1M}\,\zeta_{1M}^2+\Phi_{3M}\,\zeta_{3M}^2+\Phi_{5M}\,\zeta_{1M}\,\zeta_{3M}\right)d\varphi\,d\nu \\
& +\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1M}\left[\left(\frac{\partial\zeta_{1M}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{1M}}{\partial\nu}\right)^2\right]d\varphi\,d\nu \\
& +\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{3M}\left[\left(\frac{\partial\zeta_{3M}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{3M}}{\partial\nu}\right)^2\right]d\varphi\,d\nu \\
& +\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{5M}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\frac{\partial\zeta_{3M}}{\partial\varphi}+\Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\frac{\partial\zeta_{3M}}{\partial\nu}\right]d\varphi\,d\nu, \qquad (4.101)
\end{aligned}
$$

where Θ, x_E, x_M, s_{44M}, c_{iM} and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} ($i=1,2,3;\ j=1,3,5$) are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_{1M}, ζ_{3M}, ζ_M have the forms

$$
\begin{aligned}
& \zeta_{1M} = -\frac{p_{1n}\,\rho_{2ME}\,x_E}{\zeta_M},\ \zeta_{3M} = \frac{p_{1n}\,\rho_{2ME}\,x_E\,x_M}{\zeta_M}\left(\frac{1}{3}-\ln x_M\right), \\
& \zeta_M = x_E\left(\frac{1}{3}-\ln x_E\right)-x_M\left(\frac{1}{3}-\ln x_M\right). \qquad (4.102)
\end{aligned}
$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{1ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{2M}\neq 0$, $C_{3M}\neq 0$, $C_{1M}=0$. With regard to Equations (2.30), (2.31), (4.4)–(4.11), (2.22), (2.23), we get

$$
\varepsilon_{nM} = c_{3M}\,\zeta_{2M}\,x_n^{c_{3M}-1}, \qquad (4.103)
$$

$$
\varepsilon_{\varphi M} = \varepsilon_{\theta M} = \zeta_{2M}\,x_n^{c_{3M}-1}+\frac{\zeta_{3M}}{x_n}, \qquad (4.104)
$$

$$
\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = x_n^{c_{3M}-1}\frac{\partial\zeta_{2M}}{\partial\varphi}+\frac{1}{x_n}\frac{\partial\zeta_{3M}}{\partial\varphi}, \qquad (4.105)
$$

$$
\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = \Theta\left(x_n^{c_{3M}-1}\frac{\partial\zeta_{2M}}{\partial\nu}+\frac{1}{x_n}\frac{\partial\zeta_{3M}}{\partial\nu}\right), \qquad (4.106)
$$

$$\sigma_{nM} = \zeta_{2M}\left[(c_{1M}+c_{2M})\,c_{3M} - 2\,c_{Mm}\right] x_n^{c_{3M}-1} - \frac{\zeta_{3M}}{x_n}, \tag{4.107}$$

$$\sigma_{\varphi M} = \sigma'_{\theta M} = \zeta_{2M}\,(c_{1M} - c_{2M}\,c_{3M})\, x_n^{c_{3M}-1} + \frac{c_{1M}\,\zeta_{3M}}{x_n}, \tag{4.108}$$

$$\begin{aligned} w_M &= \kappa_{2M}\,\zeta_{2M}^2 + \kappa_{3M}\,\zeta_{3M}^2 + \kappa_{6M}\,\zeta_{2M}\,\zeta_{3M} + \frac{\chi_{2M}}{s_{44M}}\left[\left(\frac{\partial \zeta_{2M}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{2M}}{\partial \nu}\right)^2\right] \\ &+ \frac{\chi_{3M}}{s_{44M}}\left[\left(\frac{\partial \zeta_{3M}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{3M}}{\partial \nu}\right)^2\right] + \frac{\chi_{6M}}{s_{44M}}\left[\frac{\partial \zeta_{2M}}{\partial \varphi}\frac{\partial \zeta_{3M}}{\partial \varphi} + \Theta^2 \frac{\partial \zeta_{2M}}{\partial \nu}\frac{\partial \zeta_{3M}}{\partial \nu}\right], \end{aligned} \tag{4.109}$$

$$\begin{aligned} W_M &= 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{2M}\,\zeta_{2M}^2 + \Phi_{3M}\,\zeta_{3M}^2 + \Phi_{6M}\,\zeta_{2M}\,\zeta_{3M}\right) d\varphi\, d\nu \\ &+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2M}\left[\left(\frac{\partial \zeta_{2M}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{2M}}{\partial \nu}\right)^2\right] d\varphi\, d\nu \\ &+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{3M}\left[\left(\frac{\partial \zeta_{3M}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{3M}}{\partial \nu}\right)^2\right] d\varphi\, d\nu \\ &+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{6M}\left[\frac{\partial \zeta_{2M}}{\partial \varphi}\frac{\partial \zeta_{3M}}{\partial \varphi} + \Theta^2 \frac{\partial \zeta_{2M}}{\partial \nu}\frac{\partial \zeta_{3M}}{\partial \nu}\right] d\varphi\, d\nu, \end{aligned} \tag{4.110}$$

where Θ, x_E, x_M, s_{44M}, c_{iM} and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} $(i=1,2,3;\ j=2,3,6)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_{2M}, ζ_{3M}, ζ_M have the forms

$$\zeta_{2M} = -\frac{p_{1n}\,\rho_{2ME}\,x_E}{\zeta_M}, \quad \zeta_{3M} = \frac{p_{1n}\,\rho_{2ME}\,x_E\,x_M^{c_{3M}}}{\zeta_M}, \quad \zeta_M = x_E^{c_{3M}} - x_M^{c_{3M}}. \tag{4.111}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{1ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} \neq 0$. With regard to Equations (2.30), (2.31), (2.32), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\,\zeta_{1M}\left(\frac{2}{3}+\ln x_n\right)+c_{3M}\,\zeta_{2M}\,x_n^{c_{3M}-1}, \tag{4.112}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = \zeta_{1M}\left(\frac{1}{3}-\ln x_n\right)+\zeta_{2M}\,x_n^{c_{3M}-1}+\frac{\zeta_{3M}}{x_n}, \tag{4.113}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = \left(\frac{1}{3}-\ln x_n\right)\frac{\partial\zeta_{1M}}{\partial\varphi}+x_n^{c_{3M}-1}\,\frac{\partial\zeta_{2M}}{\partial\varphi}+\frac{1}{x_n}\,\frac{\partial\zeta_{3M}}{\partial\varphi}, \tag{4.114}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = \Theta\left[\left(\frac{1}{3}-\ln x_n\right)\frac{\partial\zeta_{1M}}{\partial\nu}+x_n^{c_{3M}-1}\,\frac{\partial\zeta_{2M}}{\partial\nu}+\frac{1}{x_n}\,\frac{\partial\zeta_{3M}}{\partial\nu}\right], \tag{4.115}$$

$$\begin{aligned}\sigma_{nM} = &-\,\zeta_{1M}\left[\frac{2\,(c_{1M}+2\,c_{2M})}{3}+(c_{1M}-c_{Mm})\ln x_n\right]\\&+\zeta_{2M}\left[(c_{1M}+c_{2M})\,c_{3M}-2\,c_{2M}\right]x_n^{c_{3M}-1}-\frac{2\,c_{2M}\,\zeta_{3M}}{x_n},\end{aligned} \tag{4.116}$$

$$\begin{aligned}\sigma_{\varphi M} = \sigma_{\theta M} = &\,\zeta_{1M}\left[\frac{c_{1M}+2\,c_{2M}}{3}-(c_{1M}-c_{2M})\ln x_n\right]\\&+\zeta_{2M}\,(c_{1M}-c_{2M}\,c_{3M})\,x_n^{c_{Mm}-1}+\frac{c_{1M}\,\zeta_{3M}}{x_n},\end{aligned} \tag{4.117}$$

$$\begin{aligned}w_M = &\,\kappa_{1M}\,\zeta_{1M}^2+\kappa_{2M}\,\zeta_{2M}^2+\kappa_{3M}\,\zeta_{3M}^2+\kappa_{4M}\,\zeta_{1M}\,\zeta_{2M}+\kappa_{5M}\,\zeta_{1M}\,\zeta_{3M}+\kappa_{6M}\,\zeta_{2M}\,\zeta_{3M}\\&+\frac{\chi_{1M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{1M}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{1M}}{\partial\nu}\right)^2\right]+\frac{\chi_{2M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{2M}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{2M}}{\partial\nu}\right)^2\right]\\&+\frac{\chi_{3M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{3M}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{3M}}{\partial\nu}\right)^2\right]+\frac{\chi_{4M}}{s_{44M}}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\,\frac{\partial\zeta_{2M}}{\partial\varphi}+\Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\,\frac{\partial\zeta_{2M}}{\partial\nu}\right]\\&+\frac{\chi_{5M}}{s_{44M}}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\,\frac{\partial\zeta_{3M}}{\partial\varphi}+\Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\,\frac{\partial\zeta_{3M}}{\partial\nu}\right]+\frac{\chi_{6M}}{s_{44M}}\left[\frac{\partial\zeta_{2M}}{\partial\varphi}\,\frac{\partial\zeta_{3M}}{\partial\varphi}+\Theta^2\frac{\partial\zeta_{2M}}{\partial\nu}\,\frac{\partial\zeta_{3M}}{\partial\nu}\right],\end{aligned} \tag{4.118}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{1M}\,\zeta_{1M}^2+\Phi_{2M}\,\zeta_{2M}^2+\Phi_{3M}\,\zeta_{3M}^2\right)d\varphi\,d\nu$$

$$
\begin{aligned}
&+ 4\text{¢}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} (\Phi_{4M}\,\zeta_{1M}\,\zeta_{2M} + \Phi_{5M}\,\zeta_{1M}\,\zeta_{3M} + \Phi_{6M}\,\zeta_{2M}\,\zeta_{3M})\, d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} \Psi_{1M}\left[\left(\frac{\partial\zeta_{1M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{1M}}{\partial\nu}\right)^2\right] d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} \Psi_{2M}\left[\left(\frac{\partial\zeta_{2M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2M}}{\partial\nu}\right)^2\right] d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} \Psi_{3M}\left[\left(\frac{\partial\zeta_{3M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{3M}}{\partial\nu}\right)^2\right] d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} \Psi_{4M}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\,\frac{\partial\zeta_{2M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\,\frac{\partial\zeta_{2M}}{\partial\nu}\right] d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} \Psi_{5M}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\,\frac{\partial\zeta_{3M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\,\frac{\partial\zeta_{3M}}{\partial\nu}\right] d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} \Psi_{6M}\left[\frac{\partial\zeta_{2M}}{\partial\varphi}\,\frac{\partial\zeta_{3M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{2M}}{\partial\nu}\,\frac{\partial\zeta_{3M}}{\partial\nu}\right] d\varphi\, d\nu, \qquad (4.119)
\end{aligned}
$$

where Θ, x_E, x_M, s_{44M}, c_{iM} and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} ($i = 1,2,3;\ j = 1, \ldots, 6$) are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_{iM} ($i = 1,2,3$), ζ_M have the forms

$$
\begin{aligned}
&\zeta_{1M} = -\frac{p_{1n}\,\rho_{2ME}\,x_E\,c_{3M}\,x_M^{c_{3M}-1}}{\zeta_M}, \quad \zeta_{2M} = -\frac{p_{1n}\,\rho_{2ME}\,x_e}{\zeta_M}\left(\frac{2}{3} + \ln x_M\right), \\
&\zeta_{3M} = \frac{p_1\,\rho_{2ME}\,x_E\,x_M^{c_{3m}}}{\zeta_M}\left[c_{3m}\left(\frac{1}{3} - \ln x_M\right) + \left(\frac{2}{3} + \ln x_M\right)\right], \\
&\zeta_M = c_{3m}x_M^{c_{3m}-1}\left[x_E\left(\frac{1}{3} - \ln x_E\right) - x_M\left(\frac{1}{3} - \ln r x_M\right)\right] \\
&\quad + (x_E^{c_{3m}} - x_M^{c_{3m}})\left(\frac{2}{3} + \ln x_M\right). \qquad (4.120)
\end{aligned}
$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{1ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

4.3.2 Envelope

If $\beta_{IN} \neq \beta_E = \beta_M$, then $|u_{nE}| = f(x_n)$ represents a decreasing function of $x_n \in \langle x_{IN}, x_E \rangle$, and then $C_{1E} = C_{3E} = 0$, $C_{2E} \neq 0$ (see Equation (4.4)), respectively. With regard to Equations (2.33), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\frac{p_{1n} c_{3E}}{\zeta_E} \left(\frac{x_n}{x_{IN}} \right)^{c_{3E}-1}, \tag{4.121}$$

$$\varepsilon_\varphi = \varepsilon_\theta = -\frac{p_{1n}}{\zeta_E} \left(\frac{x_n}{x_{IN}} \right)^{c_{3E}-1}, \tag{4.122}$$

$$\varepsilon_{n\varphi E} = s_{44E}\, \sigma_{n\varphi E} = -\, x_n^{c_{3E}-1} \frac{\partial}{\partial \varphi} \left(\frac{p_{1n}}{\zeta_E\, x_{IN}^{c_{3E}-1}} \right), \tag{4.123}$$

$$\varepsilon_{n\theta E} = s_{44E}\, \sigma_{n\theta E} = -\, \Theta x_n^{c_{3E}-1} \frac{\partial}{\partial \nu} \left(\frac{p_{1n}}{\zeta_E\, x_{IN}^{c_{3E}-1}} \right), \tag{4.124}$$

$$\sigma_{nE} = -\, p_{1n} \left(\frac{x_n}{x_{IN}} \right)^{c_{3E}-1}, \tag{4.125}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\frac{p_{1n}\,(c_{1E} - c_{2E}\, c_{3E})}{\zeta_E} \left(\frac{x_n}{x_{IN}^{c_{3E}-1}} \right)^{c_{3E}-1}, \tag{4.126}$$

$$\begin{aligned} w_E = {} & \kappa_{2E} \left(\frac{p_{1n}}{\zeta_E\, x_{IN}^{c_{3E}-1}} \right)^2 \\ & + \frac{\chi_{2E}}{s_{44E}} \left\{ \left[\frac{\partial C_2}{\partial \varphi} \left(\frac{p_{1n}}{\zeta_E\, x_{IN}^{c_{3E}-1}} \right) \right]^2 + \Theta^2 \left[\frac{\partial C_2}{\partial \nu} \left(\frac{p_{1n}}{\zeta_E\, x_{IN}^{c_{3E}-1}} \right) \right]^2 \right\}, \end{aligned} \tag{4.127}$$

$$\begin{aligned} W_E = {} & 4 \int_0^{\pi/2} \int_0^{\pi/2} \Phi_{2E} \left(\frac{p_{1n}}{\zeta_E\, x_{IN}^{c_{3E}-1}} \right)^2 d\varphi\, d\nu \\ & + \frac{4}{s_{44E}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{2E} \left\{ \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{1n}}{\zeta_E\, x_{IN}^{c_{3E}-1}} \right) \right]^2 + \Theta^2 \left[\frac{\partial}{\partial \nu} \left(\frac{p_{1n}}{\zeta_E\, x_{IN}^{c_{3E}-1}} \right) \right]^2 \right\} d\varphi\, d\nu, \end{aligned} \tag{4.128}$$

where Θ, x_E, s_{44E}, c_{iE} $(i=1,2,3)$ and κ_{2E}, χ_{2E}; Φ_{2E}, Ψ_{2E} are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_E has the form

$$\zeta_E = (c_{1E} + c_{2E})\, c_{3E} - 2\, c_{2E}. \tag{4.129}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} in Equation (2.27) is given by Equation (6.89). With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n = x_{IN}} = -p_{1n}\,\rho_{1ME}$, $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E} = -p_{1n}\,\rho_{2ME}$ (see Equation (4.132)), the coefficients ρ_{1ME}, ρ_{2ME} in Equations (2.27), (2.31) are derived as

$$\rho_{1ME} = \frac{1}{\zeta_E}, \quad \rho_{2ME} = \frac{1}{\zeta_E}\left(\frac{x_E}{x_{IN}}\right)^{c_{3E}-1}. \tag{4.130}$$

4.4 Condition $\beta_{IN} = \beta_E \neq \beta_M$

If $\beta_{IN} = \beta_E \neq \beta_M$, then the thermal strains and stresses in the cell matrix are determined in Section 4.2.1. The radial stress p_{2n} in Section 4.2.1 is given by Equations (2.28). The coefficients ρ_M and ρ_{2INE} in Equations (2.26) are given by Equations (3.24), (4.24), (4.34), (4.44), (4.54), (5.23), (5.33), (5.43), (5.53), (6.22), (6.32), (6.42), (6.52), (7.23), (7.33), (7.43), (7.53), (7.63), (7.73) and (3.63), (4.140), (5.129), (5.139), (6.143), respectively, with respect to minimum W_c (see Equation (2.23)).

4.4.1 Envelope

If $\beta_{IN} = \beta_E \neq \beta_M$, then $|u_{nE}| = f(x_n)$ represents an increasing function of $x_n \in \langle x_{IN}, x_E \rangle$, and then $C_{1E} \neq 0$, $C_{2E} = C_{3E} = 0$ (see Equation (4.4)), respectively. With regard to Equations (2.34), (4.4)–(4.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = \frac{p_{2n}}{\zeta_E}\left(\frac{2}{3} + \ln x_n\right), \tag{4.131}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\frac{p_{2n}}{\zeta_E}\left(\frac{1}{3} - \ln x_n\right), \tag{4.132}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\left(\frac{1}{3} - \ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_E}\right), \tag{4.133}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\Theta\left(\frac{1}{3} - \ln x_n\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_E}\right), \tag{4.134}$$

$$\sigma_{nE} = \frac{p_{2n}}{\zeta_E}\left[\frac{2\left(c_{1E} + 2c_{2E}\right)}{3} + \left(c_{1E} - c_{2E}\right)\ln x_n\right], \tag{4.135}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\frac{p_{2n}}{\zeta_E}\left[\frac{c_{1E} + 2c_{2E}}{3} - \left(c_{1E} - c_{2E}\right)\ln x_n\right], \tag{4.136}$$

$$w_E = \kappa_{1E}\left(\frac{p_{2n}}{\zeta_E}\right)^2 + \frac{\chi_{1E}}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2\right\}, \tag{4.137}$$

$$W_E = 4\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\Phi_{1E}\left(\frac{p_{2n}}{\zeta_E}\right)^2 d\varphi\, d\nu + \frac{4}{s_{44E}}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\Psi_{1E}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2\right\} d\varphi\, d\nu, \tag{4.138}$$

where Θ, x_E, s_{44E}, c_{iE} $(i=1,2,3)$ and κ_{1E}, χ_{1E}; Φ_{1E}, Ψ_{1E} are given by Equations (1.21) (1.22), (2.13), (2.18) and (4.12); (4.14), respectively, and ζ_E has the form

$$\zeta_E = \frac{2(c_{1E}+2c_{2E})}{3} + (c_{1E}-c_{2E})\ln x_E. \tag{4.139}$$

The normal stress p_{2n} is given by Equation (2.28), where ρ_M in Equation (2.28) is given by Equations (3.24), (4.24), (4.34), (4.44), (4.54), (5.23), (5.33), (5.43), (5.53), (6.22), (6.32), (6.42), (6.52), (7.23), (7.33), (7.43), (7.53), (7.63), (7.73) with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}} = -p_{2n}\,\rho_{1INE}$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E} = -p_{2n}\,\rho_{2INE}$ (see Equation (4.86)), the coefficients ρ_{1INE}, ρ_{2INE} in Equations (2.28), (2.37) are derived as

$$\rho_{1INE} = \frac{1}{\zeta_E}\left(\frac{1}{3} - \ln x_{IN}\right), \quad \rho_{2INE} = \frac{1}{\zeta_E}\left(\frac{1}{3} - \ln x_E\right). \tag{4.140}$$

Chapter 5

Mathematical Model 3

5.1 Mathematical Procedure

Let the mathematical procedure $\partial^2\text{Eq.}(2.19)/\partial x_n^2$ be performed, and then we get [2]–[23]

$$r\frac{\partial^3 u_n}{\partial x_n^3}+4x_n^2\frac{\partial^2 u_n}{\partial x_n^2}+\frac{x_n}{s_{44}\,(c_1+c_2)}\frac{\partial U_n}{\partial x_n}=0, \tag{5.1}$$

where s_{44}, c_i $(i=1,2,3)$ and $U_n=U_n\,(r,\varphi,\nu)$ are given by Equations (2.13), (2.18) and (2.21), respectively. With regard to Equations (2.20), (4.2), we get

$$x_n\frac{\partial U_n}{\partial x_n}=c_3\left(C_1\,x_n+C_2\,x_n^{c_3}+C_3\right), \tag{5.2}$$

where C_1, C_2, C_3 are integration constants, which are determined by the boundary conditions in Section 2.4. Let Equation (5.2) be substituted to Equation (5.1), and then we get

$$x_n^3\frac{\partial^3 u_n}{\partial x_n^3}+4x_n^2\frac{\partial^2 u_n}{\partial x_n^2}=C_1\,x_n^3+C_2\,x_n^{c_3}+C_3. \tag{5.3}$$

The mathematical solution of Equation (5.3), which is determined by Wronskian's method [24], is derived as

$$u_n=C_1\,x_n\left(\frac{4}{3}-\ln x_n\right)+C_2\,x_n^{c_3}+C_3\left(\frac{1}{2}+\ln x_n\right). \tag{5.4}$$

With regard to Equations (2.1)–(2.4), (2.14)–(2.17), (2.22), (5.4), we get

$$\varepsilon_n=C_1\left(\frac{1}{3}-\ln x_n\right)+C_2\,c_3\,x_n^{c_3-1}+\frac{C_3}{x_n}, \tag{5.5}$$

$$\varepsilon_\varphi=\varepsilon_\theta=C_1\left(\frac{4}{3}-\ln x_n\right)+C_2\,x_n^{c_3-1}+\frac{C_3}{x_n}\left(\frac{1}{2}+\ln x_n\right), \tag{5.6}$$

$$\varepsilon_{n\varphi}=s_{44}\,\sigma_{n\varphi}=\left(\frac{4}{3}-\ln x_n\right)\frac{\partial C_1}{\partial\varphi}+x_n^{c_3-1}\frac{\partial C_2}{\partial\varphi}+\frac{1}{x_n}\left(\frac{1}{2}+\ln x_n\right)\frac{\partial C_3}{\partial\varphi}, \tag{5.7}$$

$$\varepsilon_{n\theta} = s_{44}\,\sigma_{n\theta} = \Theta\left[\left(\frac{4}{3} - \ln x_n\right)\frac{\partial C_1}{\partial \nu} + x_n^{c_3-1}\frac{\partial C_2}{\partial \nu} + \frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial C_3}{\partial \nu}\right], \quad (5.8)$$

$$\begin{aligned}\sigma_n = {} & C_1\left[\frac{c_1 - 7c_2}{3} - (c_1 - c_2)\ln x_n\right] + C_2\left[(c_1 + c_2)\,c_3 - 2\,c_2\right]x_n^{c_3-1} \\ & + \frac{C_3}{x_n}(c_1 - 2\,c_2 \ln x_n), \end{aligned} \quad (5.9)$$

$$\begin{aligned}\sigma_\varphi = \sigma_\theta = {} & C_1\left[\frac{4c_1 - c_2}{3} - (c_1 - c_2)\ln x_n\right] + C_2\,(c_1 - c_2\,c_3)\,x_n^{c_3-1} \\ & + \frac{C_3}{x_n}\left(\frac{c_1 - 2\,c_2}{2} + c_1 \ln x_n\right), \end{aligned} \quad (5.10)$$

$$\begin{aligned} w = {} & C_1^2\,\kappa_1 + C_2^2\,\kappa_2 + C_3^2\,\kappa_3 + C_1 C_2\,\kappa_4 + C_1 C_3\,\kappa_5 + C_2 C_3\,\kappa_6 \\ & + \frac{\chi_1}{s_{44}}\left[\left(\frac{\partial C_1}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial C_1}{\partial \nu}\right)^2\right] + \frac{\chi_2}{s_{44}}\left[\left(\frac{\partial C_2}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial C_2}{\partial \nu}\right)^2\right] \\ & + \frac{\chi_3}{s_{44}}\left[\left(\frac{\partial C_3}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial C_3}{\partial \nu}\right)^2\right] + \frac{\chi_4}{s_{44}}\left(\frac{\partial C_1}{\partial \varphi}\frac{\partial C_2}{\partial \varphi} + \Theta^2\frac{\partial C_1}{\partial \nu}\frac{\partial C_2}{\partial \nu}\right) \\ & + \frac{\chi_5}{s_{44}}\left(\frac{\partial C_1}{\partial \varphi}\frac{\partial C_3}{\partial \varphi} + \Theta^2\frac{\partial C_1}{\partial \nu}\frac{\partial C_3}{\partial \nu}\right) + \frac{\chi_6}{s_{44}}\left(\frac{\partial C_2}{\partial \varphi}\frac{\partial C_3}{\partial \varphi} + \Theta^2\frac{\partial C_2}{\partial \nu}\frac{\partial C_3}{\partial \nu}\right), \end{aligned} \quad (5.11)$$

where Θ is given by Equation (1.21), and κ_i, χ_i $(i = 1, \ldots, 6)$ are derived as

$$\begin{aligned} \kappa_1 &= \frac{c_2 - c_1}{2}\ln^2 x_n + \frac{c_1 - c_2}{3}\ln x_n + \frac{17\,c_1 + c_2}{18}, \\ \kappa_2 &= \left[\frac{c_3^2\,(c_1 + c_2)}{2} + c_1\,(1 - 2\,c_3)\right]x_n^{2(c_3-1)}, \\ \kappa_3 &= \frac{c_1 \ln^2 x_n}{x_n^2} - \frac{c_1 \ln x_n}{x_n^2} + \frac{c_2 - 2\,c_1}{4x_n^2}, \\ \kappa_4 &= c_3\,(c_1 - c_2)\,x_n^{c_3-1}\ln x_n + \left[2\,c_1 + \frac{c_3\,(c_2 - 7c_1)}{3}\right]x_n^{c_3-1}, \\ \kappa_5 &= (3\,c_1 - c_2)\frac{\ln x_n}{x_n} - \frac{4\,c_1 - c_2}{3\,x_n}, \\ \kappa_6 &= 2\,c_1\,(1 - c_3)\,x_n^{c_3-2}\ln x_n + (c_2\,c_3 - c_1)\,x_n^{c_3-2}, \\ \chi_1 &= \ln^2 x_n - \frac{8}{3}\ln x_n + \frac{16}{9}, \quad \chi_2 = x_n^{2(c_3-1)}, \end{aligned}$$

$$\chi_3 = \frac{\ln^2 x_n}{x_n^2} + \frac{\ln x_n}{x_n^2} + \frac{1}{4x_n^2}, \quad \chi_4 = \frac{8}{3} x_n^{c_3-1} - 2x_n^{c_3-1} \ln x_n,$$
$$\chi_5 = \frac{4}{3x_n} + \frac{5\ln x_n}{3x_n} - \frac{2\ln^2 x_n}{x_n}, \quad \chi_6 = 2x_n^{c_3-2} \ln x_n + x_n^{c_3-2}. \tag{5.12}$$

With regard to Equations (4.14), (5.12), we get

$$\Phi_{1q} = \frac{c_{2q} - c_{1q}}{6} \left\{ x_{2n}^3 \left[\left(\ln x_{2n} - \frac{1}{3} \right)^2 + \frac{1}{9} \right] - x_{1n}^3 \left[\left(\ln x_{1n} - \frac{1}{3} \right)^2 + \frac{1}{9} \right] \right\}$$
$$+ \frac{c_{1q} - c_{2q}}{9} \left[x_{2n}^3 \left(\ln x_{2n} - \frac{1}{3} \right) - x_{1n}^3 \left(\ln x_{1n} - \frac{1}{3} \right) \right] + \frac{17\, c_{1q} + c_{2q}}{54} \left(x_{2n}^3 - x_{1n}^3 \right),$$
$$\Phi_{2q} = \frac{1}{2\, c_{3q} + 1} \left[\frac{c_{3q}^2 \left(c_{1q} + c_{2q} \right)}{2} + c_{1q} \left(1 - 2\, c_{3q} \right) \right] \left(x_{2n}^{2\, c_{3q}+1} - x_{1n}^{2\, c_{3q}+1} \right),$$
$$\Phi_{3q} = c_{1q} \left[x_{2n} \left(\ln^2 x_{2n} - 2\ln x_{2n} + 2 \right) - x_{1n} \left(\ln^2 x_{1n} - 2\ln x_{1n} + 2 \right) \right]$$
$$- c_{1q} \left[x_{2n} \left(\ln x_{2n} - 1 \right) - x_{1n} \left(\ln x_{1n} - 1 \right) \right] + \frac{c_{2q} - 2\, c_{1q}}{4} \left(x_{2n} - x_{1n} \right),$$
$$\Phi_{4q} = \frac{c_{3q} \left(c_{1q} - c_{2q} \right)}{c_{3q} + 2} \left[x_{2n}^{c_{3q}+2} \left(\ln x_{2n} - \frac{1}{c_{3q} + 2} \right) - x_{1n}^{c_{3q}+2} \left(\ln x_{1n} - \frac{1}{c_{3q} + 2} \right) \right]$$
$$+ \frac{1}{c_{3q} + 2} \left[2\, c_{1q} + \frac{c_{3q} \left(c_{2q} - 7c_{1q} \right)}{3} \right] \left(x_{2n}^{c_{3q}+2} - x_{1n}^{c_{3q}+2} \right),$$
$$\Phi_{5q} = \frac{3\, c_{1q} - c_{2q}}{2} \left[x_{2n}^2 \left(\ln x_{2n} - \frac{1}{2} \right) - x_{1n}^2 \left(\ln x_{1n} - \frac{1}{2} \right) \right]$$
$$- \frac{4\, c_{1q} - c_{2q}}{6} \left(x_{2n}^2 - x_{1n}^2 \right),$$
$$\Phi_{6q} = \frac{2\, c_{1q} \left(1 - c_{3q} \right)}{c_{3q} + 1} \left[x_{2n}^{c_{3q}+1} \left(\ln x_{2n} - \frac{1}{c_3 + 1} \right) - x_{1n}^{c_{3q}+1} \left(\ln x_{1n} - \frac{1}{c_{3q} + 1} \right) \right]$$
$$+ \frac{c_{2q}\, c_{3q} - c_{1q}}{c_{3q} + 1} \left(x_{2n}^{c_{3q}+1} - x_{1n}^{c_{3q}+1} \right),$$
$$\Psi_{1q} = \frac{x_{2n}^3}{3} \left[\left(\ln x_{2n} - 3 \right) \left(\ln x_{2n} - \frac{1}{3} \right) + \frac{17}{9} \right]$$
$$- \frac{x_{1n}^3}{3} \left[\left(\ln x_{1n} - 3 \right) \left(\ln x_{1n} - \frac{1}{3} \right) + \frac{17}{9} \right],$$
$$\Psi_{2q} = \frac{x_{2n}^{2\, c_{3q}+1} - x_{1n}^{2\, c_{3q}+1}}{2\, c_{3q} + 1},$$

$$
\begin{aligned}
\Psi_{3q} &= x_{2n} \ln x_{2n} \left(\ln x_{2n} - 1\right) - x_{1n} \ln x_{1n} \left(\ln x_{1n} - 1\right) + \frac{5\left(x_{2n} - x_{1n}\right)}{4}, \\
\Psi_{4q} &= \frac{2}{c_{3q}+2} \left\{ x_{2n}^{c_{3q}+2} \left[\frac{4c_{3q}+11}{3\left(c_{3q}+2\right)} - \ln x_{2n} \right] - x_{1n}^{c_{3q}+2} \left[\frac{4c_{3q}+11}{3\left(c_{3q}+2\right)} - \ln x_{1n} \right] \right\}, \\
\Psi_{5q} &= \frac{2\left(x_{2n}^2 - x_{1n}^2\right)}{3} + \frac{5}{6} \left[x_{2n}^2 \left(\ln x_{2n} - \frac{1}{2} \right) - x_{1n}^2 \left(\ln x_{1n} - \frac{1}{2} \right) \right] \\
&\quad - x_{2n}^2 \left(\ln^2 x_{2n} - \ln x_{2n} + \frac{1}{2} \right) + x_{1n}^2 \left(\ln^2 x_{1n} - \ln x_{1n} + \frac{1}{2} \right), \\
\Psi_{6q} &= \frac{2}{c_{3q}+1} \left[x_{2n}^{c_{3q}+1} \left(\ln x_{2n} - \frac{1}{c_{3q}+1} \right) - x_{1n}^{c_{3q}+1} \left(\ln x_{1n} - \frac{1}{c_{3q}+1} \right) \right] \\
&\quad + \frac{1}{c_{3q}+1} \left(x_{2n}^{c_{3q}+1} - x_{1n}^{c_{3q}+1} \right), \\
q &= E \quad \Rightarrow \quad x_{1n} = x_{IN},\ x_{2n} = x_E, \\
q &= M \quad \Rightarrow \quad x_{1n} = x_E,\ x_{2n} = x_M,
\end{aligned}
\tag{5.13}
$$

where x_{IN}, x_E, x_M are given by Equations (1.22), respectively. In case of the ellipsoidal inclusion, we get $(u_{nIN})_{x_n \to 0} \longrightarrow \pm\infty$, $(\varepsilon_{IN})_{x_n \longrightarrow 0} \longrightarrow \pm\infty$, $(\sigma_{IN})_{r \to 0} \longrightarrow \pm\infty$ due to $(\ln x_n)_{x_n \to 0} \longrightarrow \pm\infty$ and $(x_n^{c_3})_{x_n \to 0} \longrightarrow \pm\infty$ for $c_3 < 0$ (see Equations (2.18), (5.4)–(5.10)). Accordingly, the mathematical solutions (5.4)–(5.10) are suitable for the envelope and matrix.

Additionally, the function $f = x_n\left[(4/3) - \ln x_n\right]$ is increasing or decreasing for $x_n \leq 0.513$ m or $x_n \geq 1.396$ m, respectively. With regard to dimensions of components of a real three-component material, $f = x_n\left[(4/3) - \ln x_n\right]$ represents an increasing function of x_n. Finally, $f = (1/2) + \ln x_n$ represents an increasing function of x_n.

5.2 Condition $\beta_{IN} \neq \beta_E \neq \beta_M$

5.2.1 Matrix

The integration constants C_{1M}, C_{2M}, C_{3M} for the matrix (see Equation (4.4)) are determined by the boundary conditions (2.29), (2.30) or 2.29), (2.30) (2.32). The boundary conditions result in the following combinations of C_{1M}, C_{2M}, C_{3M}. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$. With regard to Equations (2.29), (2.30), (5.4)–(5.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M}\left[\frac{1}{3} - \ln x_n - c_{3M}\left(\frac{4}{3} - \ln x_M\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{5.14}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\frac{4}{3} - \ln x_n - \left(\frac{4}{3} - \ln x_M\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{5.15}$$

$$\begin{aligned}\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = &\left(\ln x_n - \frac{4}{3}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\\ &+ x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\left(\frac{4}{3} - \ln x_M\right)\right],\end{aligned} \tag{5.16}$$

$$\begin{aligned}\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = \Theta\Bigg\{&\left(\ln x_n - \frac{4}{3}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\\ &+ x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\left(\frac{4}{3} - \ln x_M\right)\right]\Bigg\},\end{aligned} \tag{5.17}$$

$$\begin{aligned}\sigma_{nM} = \frac{p_{2n}}{\zeta_M}\Bigg\{&\frac{7c_{2M} - c_{1M}}{3} + (c_{1M} - c_{2M})\ln x_n\\ &+ [(c_{1M} + c_{2M})\,c_{3M} - 2c_{2M}]\left(\frac{4}{3} - \ln x_M\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\Bigg\},\end{aligned} \tag{5.18}$$

$$\begin{aligned}\sigma_{\varphi M} = \sigma_{\theta M} = \frac{p_{2n}}{\zeta_M}\Bigg[&\frac{c_{2M} - 4c_{1M}}{3} + (c_{1M} - c_{2M})\ln x_n\\ &+ (c_{1M} - c_{2M}\,c_{3M})\left(\frac{4}{3} - \ln x_M\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\Bigg],\end{aligned} \tag{5.19}$$

$$\begin{aligned}w_M = &\left(\frac{p_{2n}}{\zeta_M}\right)^2\left\{\kappa_{1M} + \kappa_{2M}\left(\frac{4 - 3\ln x_M}{3x_M^{c_{3M}-1}}\right)^2 + \frac{\kappa_{4M}\,(3\ln x_M - 4)}{3x_M^{c_{3M}-1}}\right\}\\ &+ \frac{\chi_{1M}}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2\right\}\\ &+ \frac{\chi_{2M}}{s_{44M}}\left(\left\{\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\,(4 - 3\ln x_M)}{3\,\zeta_M\, x_M^{c_{3M}-1}}\right]\right\}^2 + \Theta^2\left\{\frac{\partial}{\partial\nu}\left[\frac{p_{2n}\,(4 - 3\ln x_M)}{3\,\zeta_M\, x_M^{c_{3M}-1}}\right]\right\}^2\right)\end{aligned}$$

$$+ \frac{\chi_{4M}}{s_{44M}} \frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\,(3\ln x_M - 4)}{3\,\zeta_M\, x_M^{c_{3M}-1}}\right]$$

$$+ \frac{\chi_{4M}\,\Theta^2}{s_{44M}} \frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left[\frac{p_{2n}\,(3\ln x_M - 4)}{3\,\zeta_M\, x_M^{c_{3M}-1}}\right], \tag{5.20}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\frac{p_{2n}}{\zeta_M}\right)^2\left\{\Phi_{1M} + \Phi_{2M}\left(\frac{4-3\ln x_M}{3x_M^{c_{3M}-1}}\right)^2 + \frac{\Phi_{4M}\,(3\ln x_M - 4)}{3x_M^{c_{3M}-1}}\right\} d\varphi\, d\nu$$

$$+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1M}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2\right\} d\varphi\, d\nu$$

$$+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2M}\left(\left\{\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\,(4-3\ln x_M)}{3\,\zeta_M\, x_M^{c_{3M}-1}}\right]\right\}^2\right) d\varphi\, d\nu$$

$$+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2M}\,\Theta^2\left(\left\{\frac{\partial}{\partial\nu}\left[\frac{p_{2n}\,(4-3\ln x_M)}{3\,\zeta_M\, x_M^{c_{3M}-1}}\right]\right\}^2\right) d\varphi\, d\nu$$

$$+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{4M} \frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}\,(3\ln x_M - 4)}{3\,\zeta_M\, x_M^{c_{3M}-1}}\right] d\varphi\, d\nu$$

$$+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{4M}\,\Theta^2 \frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left[\frac{p_{2n}\,(3\ln x_M - 4)}{3\,\zeta_M\, x_M^{c_{3M}-1}}\right] d\varphi\, d\nu, \tag{5.21}$$

where Θ, x_M, s_{44M}, c_{iM} $(i = 1,2,3)$ and κ_{jM}, χ_{jM}; Φ_{jM}, Ψ_{jM} $(j = 1,2,4)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.13); (4.15), respectively, and ζ_M, ζ_{iM} $(i = 1,2)$ have the forms

$$\zeta_M = \zeta_{2M} - \zeta_{1M}\left(\frac{4}{3} - \ln x_M\right), \quad \zeta_{1M} = [(c_{1M} + c_{2M})\,c_{3M} - 2\,c_{2M}]\left(\frac{x_E}{x_M}\right)^{c_{3M}-1},$$

$$\zeta_{2M} = \frac{c_{1M} - 7c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_E, \tag{5.22}$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i, j = 1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)).

With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E} = -p_{2n}\rho_M$ and Equation (5.15), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M = \frac{1}{\zeta_M}\left[\frac{4}{3} - \ln x_E - \left(\frac{4}{3} - \ln x_M\right)\left(\frac{x_E}{x_M}\right)^{c_{3M}-1}\right]. \tag{5.23}$$

Conditions $C_{1M} \neq 0$, $C_{3M} \neq 0$, $C_{2M} = 0$. With regard to Equations (2.29), (2.30), (5.4)–(5.11), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M}\left[\left(\frac{1}{2} + \ln x_M\right)\left(\frac{1}{3} - \ln x_n\right) - \frac{x_M}{x_n}\left(\frac{4}{3} - \ln x_M\right)\right], \tag{5.24}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\left(\frac{1}{2} + \ln x_M\right)\left(\frac{4}{3} - \ln x_n\right) - \frac{x_M}{x_n}\left(\frac{4}{3} - \ln x_M\right)\left(\frac{1}{2} + \ln x_n\right)\right], \tag{5.25}$$

$$\begin{aligned}\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = {} & \left(\ln x_n - \frac{4}{3}\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2} + \ln x_M\right)\right] \\ & + \frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\ln x_M - \frac{4}{3}\right)\right],\end{aligned} \tag{5.26}$$

$$\begin{aligned}\varepsilon_{n\nu M} = s_{44M}\,\sigma_{n\varphi M} = \Theta\Bigg\{ & \left(\ln x_n - \frac{4}{3}\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2} + \ln x_M\right)\right] \\ & + \frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\ln x_M - \frac{4}{3}\right)\right]\Bigg\},\end{aligned} \tag{5.27}$$

$$\begin{aligned}\sigma_{nM} = \frac{p_{2n}}{\zeta_M}\Bigg\{ & \left(\frac{1}{2} + \ln x_M\right)\left[\frac{c_{1M} - 7c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right] \\ & + \frac{x_M}{x_n}\left(\frac{4}{3} - \ln x_M\right)(c_{1M} - 2c_{2M}\ln x_n)\Bigg\},\end{aligned} \tag{5.28}$$

$$\begin{aligned}\sigma_{\varphi M} = \sigma_{\theta M} = \frac{p_{2n}}{\zeta_M}\Bigg\{ & \left(\frac{1}{2} + \ln x_M\right)\left[\frac{4c_{1M} - c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right] \\ & + \frac{x_M}{x_n}\left(\frac{4}{3} - \ln x_M\right)\left(\frac{c_{1M} - c_{2M}}{2} + c_{3M}\ln x_n\right)\Bigg\},\end{aligned} \tag{5.29}$$

$$w_M = \left(\frac{p_{2n}}{\zeta_M}\right)^2\left[\kappa_{1M}\left(\frac{1}{2} + \ln x_M\right) + \kappa_{3M}x_M^2\left(\frac{4}{3} - \ln x_M\right)^2\right.$$

$$
\begin{aligned}
&+ \kappa_{5M}\, x_M \left(\frac{1}{2}+\ln x_M\right)\left(\frac{4}{3}-\ln x_M\right)\Bigg] \\
&+ \frac{\chi_{1M}}{s_{44M}}\left(\left\{\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2}+\ln x_M\right)\right]\right\}^2 + \Theta^2\left\{\frac{\partial}{\partial\nu}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2}+\ln x_M\right)\right]\right\}^2\right) \\
&+ \frac{\chi_{3M}}{s_{44M}}\left(\left\{\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\frac{4}{3}-\ln x_M\right)\right]\right\}^2 + \Theta^2\left\{\frac{\partial}{\partial\nu}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\frac{4}{3}-\ln x_M\right)\right]\right\}^2\right) \\
&+ \frac{\chi_{5M}}{s_{44M}}\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2}+\ln x_M\right)\right]\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\frac{4}{3}-\ln x_M\right)\right] \\
&+ \frac{\chi_{5M}\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2}+\ln x_M\right)\right]\frac{\partial}{\partial\nu}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\frac{4}{3}-\ln x_M\right)\right],
\end{aligned}
\tag{5.30}
$$

$$
\begin{aligned}
W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}&\left(\frac{p_{2n}}{\zeta_M}\right)^2\Bigg[\Phi_{1M}\left(\frac{1}{2}+\ln x_M\right)+\Phi_{3M}x_M^2\left(\frac{4}{3}-\ln x_M\right)^2 \\
&+ \Phi_{5M}x_M\left(\frac{1}{2}+\ln x_M\right)\left(\frac{4}{3}-\ln x_M\right)\Bigg]\,d\varphi\,d\nu \\
+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}&\Psi_{1M}\left(\left\{\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2}+\ln x_M\right)\right]\right\}^2\right. \\
&\left.+ \Theta^2\left\{\frac{\partial}{\partial\nu}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2}+\ln x_M\right)\right]\right\}^2\right)\,d\varphi\,d\nu \\
+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}&\left(\Psi_{3M}\left\{\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\frac{4}{3}-\ln x_M\right)\right]\right\}^2\right. \\
&\left.+ \Theta^2\left\{\frac{\partial}{\partial\nu}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\frac{4}{3}-\ln x_M\right)\right]\right\}^2\right)\,d\varphi\,d\nu \\
+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}&\Psi_{5M}\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2}+\ln x_M\right)\right]\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\frac{4}{3}-\ln x_M\right)\right]\,d\varphi\,d\nu \\
+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}&\Phi_{5M}\Theta^2\frac{\partial}{\partial\nu}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2}+\ln x_M\right)\right]\frac{\partial}{\partial\nu}\left[\frac{p_{2n}x_M}{\zeta_M}\left(\frac{4}{3}-\ln x_M\right)\right]\,d\varphi\,d\nu,
\end{aligned}
\tag{5.31}
$$

where Θ, x_M, s_{44M}, c_{iM} $(i = 1,2,5)$ and κ_{jM}, χ_{jM}; Φ_{jM}, Ψ_{jM} $(j = 1,3,5)$ are given by

Equations (1.21), (1.22), (2.13), (2.18) and (4.13); (4.15), respectively, and ζ_M, ζ_{iM} $(i = 1{,}2)$ have the forms

$$\zeta_M = \frac{\zeta_{2M}}{x_M}\left(\frac{1}{2} + \ln x_M\right) - \zeta_{1M}\left(\frac{4}{3} - \ln x_M\right), \quad \zeta_{1M} = \frac{x_M}{x_E}\left(c_{1M} - 2c_{2M}\ln x_E\right),$$
$$\zeta_{2M} = x_M\left[\frac{c_{1M} - 7c_{2M}}{3} - \left(c_{1M} - c_{2M}\right)\ln x_E\right]. \tag{5.32}$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i, j = 1{,}2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n = x_E} = -p_{2n}\rho_M$ and Equation (5.25), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M = \frac{1}{\zeta_M}\left[\left(\frac{1}{2} + \ln x_M\right)\left(\frac{4}{3} - \ln x_E\right) - \frac{x_M}{x_E}\left(\frac{4}{3} - \ln x_M\right)\left(\frac{1}{2} + \ln x_E\right)\right]. \tag{5.33}$$

Conditions $C_{2M} \neq 0$, $C_{3M} \neq 0$, $C_{1M} = 0$. With regard to Equations (2.29), (2.30), (5.4)–(5.11), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M}\left[c_{3M}\left(\frac{1}{2} + \ln x_M\right)x_n^{c_{3M}-1} - \frac{x_M^{c_{3M}}}{x_n}\right], \tag{5.34}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\left(\frac{1}{2} + \ln x_M\right)x_n^{c_{3M}-1} - \frac{x_M^{c_{3M}}}{x_n}\left(\frac{1}{2} + \ln x_n\right)\right], \tag{5.35}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = \frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M^{c_{3M}}}{\zeta_M}\right) - x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2} + \ln x_M\right)\right], \tag{5.36}$$

$$\varepsilon_{n\nu M} = s_{44M}\,\sigma_{n\nu M} = \Theta\left\{\frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M^{c_{3M}}}{\zeta_M}\right) - x_n^{c_{3M}-1}\frac{\partial}{\partial\nu}\left[\frac{p_{2n}}{\zeta_M}\left(\frac{1}{2} + \ln x_M\right)\right]\right\}, \tag{5.37}$$

$$\sigma_{nM} = -\frac{p_{2n}x_n^{c_{3M}-1}}{\zeta_M} \times\left\{\left[c_{3M}\left(c_{1M} + c_{2M}\right) - 2c_{2M}\right]\left(\frac{1}{2} + \ln x_M\right) - \frac{x_M}{x_n}\left(c_{1M} - 2c_{2M}\ln x_n\right)\right\}, \tag{5.38}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} =$$
$$- \frac{p_{2n} x_n^{c_{3M}-1}}{\zeta_M} \left[(c_{1M} - c_{2M} c_{3M}) \left(\frac{1}{2} + \ln x_M \right) - \frac{x_M}{x_n} \left(\frac{c_{1M} - 2c_{2M}}{2} + c_{1M} \ln x_n \right) \right], \tag{5.39}$$

$$\begin{aligned} w_M = {} & \left(\frac{p_{2n}}{\zeta_M} \right)^2 \left[\kappa_{2M} \left(\frac{1}{2} + \ln x_M \right)^2 + \kappa_{3M} x_M^{2c_{3M}} + \kappa_{6M} x_M^{c_{3M}} \left(\frac{1}{2} + \ln x_M \right) \right] \\ & + \frac{\chi_{2M}}{s_{44M}} \left\{ \frac{\partial}{\partial \varphi} \left[\frac{p_{2n}}{\zeta_M} \left(\frac{1}{2} + \ln x_M \right) \right]^2 + \Theta^2 \frac{\partial}{\partial \nu} \left[\frac{p_{2n}}{\zeta_M} \left(\frac{1}{2} + \ln x_M \right) \right]^2 \right\} \\ & + \frac{\chi_{3M}}{s_{44M}} \left[\frac{\partial C_{3M}}{\partial \varphi} \left(\frac{p_{2n} x_M^{c_{3M}}}{\zeta_M} \right)^2 + \Theta^2 \frac{\partial C_{3M}}{\partial \nu} \left(\frac{p_{2n} x_M^{c_{3M}}}{\zeta_M} \right)^2 \right] \\ & - \frac{\chi_{6M} \Theta}{s_{44M}} \left\{ \frac{\partial}{\partial \varphi} \left[\frac{p_{2n}}{\zeta_M} \left(\frac{1}{2} + \ln x_M \right) \right] \frac{\partial}{\partial \varphi} \left(\frac{p_{2n} x_M^{c_{3M}}}{\zeta_M} \right) \right. \\ & \qquad \left. + \frac{\partial}{\partial \nu} \left[\frac{p_{2n}}{\zeta_M} \left(\frac{1}{2} + \ln x_M \right) \right] \frac{\partial}{\partial \nu} \left(\frac{p_{2n} x_M^{c_{3M}}}{\zeta_M} \right) \right\}, \end{aligned} \tag{5.40}$$

$$\begin{aligned} W_M = {} & 4 \int_0^{\pi/2} \int_0^{\pi/2} \left(\frac{p_{2n}}{\zeta_M} \right)^2 \left[\Phi_{2M} \left(\frac{1}{2} + \ln x_M \right)^2 + \Phi_{3M} x_M^{2c_{3M}} \right. \\ & \qquad \left. + \Phi_{6M} x_M^{c_{3M}} \left(\frac{1}{2} + \ln x_M \right) \right] d\varphi \, d\nu \\ & + \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{2M} \left\{ \frac{\partial}{\partial \varphi} \left[\frac{p_{2n}}{\zeta_M} \left(\frac{1}{2} + \ln x_M \right) \right]^2 + \Theta^2 \frac{\partial}{\partial \nu} \left[\frac{p_{2n}}{\zeta_M} \left(\frac{1}{2} + \ln x_M \right) \right]^2 \right\} d\varphi \, d\nu \\ & + \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{3M} \left[\frac{\partial C_{3M}}{\partial \varphi} \left(\frac{p_{2n} x_M^{c_{3M}}}{\zeta_M} \right)^2 + \Theta^2 \frac{\partial C_{3M}}{\partial \nu} \left(\frac{p_{2n} x_M^{c_{3M}}}{\zeta_M} \right)^2 \right] d\varphi \, d\nu \\ & - \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{6M} \Theta \left\{ \frac{\partial}{\partial \varphi} \left[\frac{p_{2n}}{\zeta_M} \left(\frac{1}{2} + \ln x_M \right) \right] \frac{\partial}{\partial \varphi} \left(\frac{p_{2n} x_M^{c_{3M}}}{\zeta_M} \right) \right. \\ & \qquad \left. + \frac{\partial}{\partial \nu} \left[\frac{p_{2n}}{\zeta_M} \left(\frac{1}{2} + \ln x_M \right) \right] \frac{\partial}{\partial \nu} \left(\frac{p_{2n} x_M^{c_{3M}}}{\zeta_M} \right) \right\} d\varphi \, d\nu, \end{aligned} \tag{5.41}$$

where Θ, x_M, s_{44M}, c_{iM} ($i = 1,2,3$) and κ_{jM}, χ_{jM}; Φ_{jM}, Ψ_{jM} ($j = 2,3,6$) are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.13); (4.15), respectively, and ζ_M, ζ_{iM}

($i=1{,}2$) have the forms

$$\zeta_M = \frac{\zeta_{2M}}{x_M}\left(\frac{1}{2}+\ln x_M\right) - \zeta_{1M}\, x_M^{c_{3M}-1}, \quad \zeta_{1M} = \frac{x_M}{x_E}\left(c_{1M} - 2c_{2M}\ln x_E\right),$$
$$\zeta_{2M} = x_M\left[c_{3M}\left(c_{1M}+c_{2M}\right) - 2c_{2M}\right] x_E^{c_{3M}-1}. \tag{5.42}$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} ($i,j=1{,}2$) in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E} = -p_{2n}\rho_M$ and Equation (5.35), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M = \frac{1}{\zeta_M}\left[\left(\frac{1}{2}+\ln x_M\right) x_E^{c_{3M}-1} - \frac{x_M^{c_{3M}}}{x_E}\left(\frac{1}{2}+\ln x_E\right)\right]. \tag{5.43}$$

Conditions $C_{1M}\neq 0$**,** $C_{2M}\neq 0$**,** $C_{3M}\neq 0$**.** With regard to Equations (2.29)–(2.32), (5.4)–(5.11), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M}\left[\zeta_{1M}\left(\frac{1}{3}-\ln x_n\right) + \zeta_{2M}\, c_{3M}\, x_n^{c_{3M}-1} + \frac{\zeta_{3M}}{x_n}\right], \tag{5.44}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\zeta_{1M}\left(\frac{4}{3}-\ln x_n\right) + \zeta_{2M}\, x_n^{c_{3M}-1} + \frac{\zeta_{3M}}{x_n}\left(\frac{1}{2}+\ln x_n\right)\right], \tag{5.45}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left[\left(\frac{4}{3}-\ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right) + x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\right.$$
$$\left. + \frac{1}{x_n}\left(\frac{1}{2}+\ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right], \tag{5.46}$$

$$\varepsilon_{n\nu M} = s_{44M}\,\sigma_{n\varphi M} = -\Theta\left[\left(\frac{4}{3}-\ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right) + x_n^{c_{3M}-1}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\right.$$
$$\left. + \frac{1}{x_n}\left(\frac{1}{2}+\ln x_n\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right], \tag{5.47}$$

$$\sigma_{nM} = -\frac{p_{2n}}{\zeta_M}\left\{\zeta_{1M}\left[\frac{c_{1M}-7c_{2M}}{3} - \left(c_{1M}-c_{2M}\right)\ln x_n\right]\right.$$
$$\left. + \zeta_{2M}\left[\left(c_{1M}+c_{2M}\right)c_{3M} - 2c_{2M}\right] x_n^{c_{3M}-1} + \frac{\zeta_{3M}\left(c_{1M}-2c_{2M}\ln x_n\right)}{x_n}\right\}, \tag{5.48}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left\{\zeta_{1M}\left[\frac{4c_{1M} - c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right]\right.$$
$$\left. + \zeta_{2M}(c_{1M} - c_{2M}c_{3M})x_n^{c_{3M}-1} + \zeta_{3M}\left(\frac{c_{1M} - 2c_{2M}}{2} + c_{1M}\ln x_n\right)\right\}, \quad (5.49)$$

$$\begin{aligned}
w_M = &\left(\frac{p_{2n}}{\zeta_M}\right)^2 \left(\kappa_{1M}\zeta_{1M}^2 + \kappa_{2M}\zeta_{2M}^2 + \kappa_{3M}\zeta_{3M}^2\right. \\
&\left. + \kappa_{4M}\zeta_{1M}\zeta_{2M} + \kappa_{5M}\zeta_{1M}\zeta_{3M} + \kappa_{6M}\zeta_{2M}\zeta_{3M}\right) \\
&+ \frac{\chi_{1M}}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\right]^2\right\} \\
&+ \frac{\chi_{2M}}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\right]^2\right\} \\
&+ \frac{\chi_{3M}}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)\right]^2\right\} \\
&+ \frac{\chi_{4M}}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right) \\
&+ \frac{\chi_{4M}\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right) \\
&+ \frac{\chi_{5M}}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right) \\
&+ \frac{\chi_{5M}\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{1M}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right) \\
&+ \frac{\chi_{6M}}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right) \\
&+ \frac{\chi_{6M}\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{2M}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\zeta_{3M}}{\zeta_M}\right)
\end{aligned} \quad (5.50)$$

$$\begin{aligned}
W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}&\left(\frac{p_{2n}}{\zeta_M}\right)^2\left(\Phi_{1M}\zeta_{1M}^2 + \Phi_{2M}\zeta_{2M}^2 + \Phi_{3M}\zeta_{3M}^2\right. \\
&\left. + \Phi_{4M}\zeta_{1M}\zeta_{2M} + \Phi_{5M}\zeta_{1M}\zeta_{3M} + \Phi_{6M}\zeta_{2M}\zeta_{3M}\right) d\varphi\, d\nu
\end{aligned}$$

$$
\begin{aligned}
&+ \frac{4}{s_{44M}} \int_0^{\pi/2}\int_0^{\pi/2} \Psi_{1M} \left\{ \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\zeta_{1M}}{\zeta_M} \right) \right]^2 + \Theta^2 \left[\frac{\partial}{\partial \nu} \left(\frac{p_{2n}\zeta_{1M}}{\zeta_M} \right) \right]^2 \right\} d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}} \int_0^{\pi/2}\int_0^{\pi/2} \Psi_{2M} \left\{ \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\zeta_{2M}}{\zeta_M} \right) \right]^2 + \Theta^2 \left[\frac{\partial}{\partial \nu} \left(\frac{p_{2n}\zeta_{2M}}{\zeta_M} \right) \right]^2 \right\} d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}} \int_0^{\pi/2}\int_0^{\pi/2} \Psi_{3M} \left\{ \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\zeta_{3M}}{\zeta_M} \right) \right]^2 + \Theta^2 \left[\frac{\partial}{\partial \nu} \left(\frac{p_{2n}\zeta_{3M}}{\zeta_M} \right) \right]^2 \right\} d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}} \int_0^{\pi/2}\int_0^{\pi/2} \Psi_{4M} \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\zeta_{1M}}{\zeta_M} \right) \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\zeta_{2M}}{\zeta_M} \right) \right. \\
&\qquad \left. + \Theta^2 \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\zeta_{1M}}{\zeta_M} \right) \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\zeta_{2M}}{\zeta_M} \right) \right] d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}} \int_0^{\pi/2}\int_0^{\pi/2} \Psi_{5M} \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\zeta_{1M}}{\zeta_M} \right) \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\zeta_{3M}}{\zeta_M} \right) \right. \\
&\qquad \left. + \Theta^2 \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\zeta_{1M}}{\zeta_M} \right) \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\zeta_{3M}}{\zeta_M} \right) \right] d\varphi\, d\nu \\
&+ \frac{4}{s_{44M}} \int_0^{\pi/2}\int_0^{\pi/2} \Psi_{6M} \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\zeta_{2M}}{\zeta_M} \right) \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\zeta_{3M}}{\zeta_M} \right) \right. \\
&\qquad \left. + \Theta^2 \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\zeta_{2M}}{\zeta_M} \right) \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\zeta_{3M}}{\zeta_M} \right) \right] d\varphi\, d\nu, \qquad (5.51)
\end{aligned}
$$

where Θ, x_M, s_{44M}, c_{iM} $(i=1,2,3)$ and κ_{jM}, χ_{jM}; Φ_{jM}, Ψ_{jM} $(j=1,\ldots,6)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.13); (4.15), respectively, and ζ_M, ζ_{iM} $(i=1,2,3)$ have the forms

$$
\begin{aligned}
\zeta_{1M} &= x_M^{c_{3M}-1} \left[c_{3M} \left(\frac{1}{2} + \ln x_M \right) - 1 \right], \\
\zeta_{2M} &= \frac{4}{3} - \ln x_M - \left(\frac{1}{2} + \ln x_M \right) \left(\frac{1}{3} - \ln x_M \right), \\
\zeta_{3M} &= x_M^{c_{3M}} \left[\frac{1}{3} - \ln x_M - c_{3M} \left(\frac{4}{3} - \ln x_M \right) \right], \\
\zeta_M &= x_M^{c_{3M}-1} \left[\frac{c_{1M} - 7c_{2M}}{3} - (c_{1M} - c_{2M}) \ln x_E \right]
\end{aligned}
$$

$$
\begin{aligned}
&+ \left[(c_{1M}+c_{2M})c_{3M}-2c_{2M}\right]\left(\frac{1}{2}+\ln x_M\right)\left(\frac{1}{3}-\ln x_M\right)x_E^{c_{3M}-1}\\
&+ (c_{1M}-2c_{2M}\ln x_E)\left(\frac{4}{3}-\ln x_M\right)\frac{c_{3M}x_M^{c_{3M}}}{x_E}\\
&-\left\{c_{3M}\left[\frac{c_{1M}-7c_{2M}}{3}-(c_{1M}-c_{2M})\ln x_E\right]\left(\frac{1}{2}+\ln x_M\right)x_M^{c_{3M}-1}\right.\\
&+\left[(c_{1M}+c_{2M})c_{3M}-2c_{2M}\right]\left(\frac{4}{3}-\ln x_M\right)x_E^{c_{3M}-1}\\
&\left.+\frac{(c_{1M}-2c_{2M}\ln x_E)x_M^{c_{3M}}}{x_E}\left(\frac{1}{3}-\ln x_M\right)\right\}. \qquad (5.52)
\end{aligned}
$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i,j=1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E}=-p_{2n}\rho_M$ and Equation (5.45), the coefficient ρ_M in Equation (2.26) is derived as

$$
\rho_M=\frac{1}{\zeta_M}\left[\zeta_{1M}\left(\frac{4}{3}-\ln x_E\right)+\zeta_{2M}x_E^{c_{3M}-1}+\frac{\zeta_{3M}}{x_E}\left(\frac{1}{2}+\ln x_E\right)\right]. \qquad (5.53)
$$

5.2.2 Envelope

The integration constants C_{1E}, C_{2E}, C_{3E} for the envelope (see Equation (5.4)) are determined by the boundary conditions (2.33), (2.34). The boundary conditions result in the following combinations of C_{1E}, C_{2E}, C_{3E}, i.e., $C_{1E}\neq 0$, $C_{2E}\neq 0$, $C_{3E}=0$; $C_{1E}\neq 0$, $C_{3E}\neq 0$, $C_{2E}=0$; $C_{2E}\neq 0$, $C_{3E}\neq 0$, $C_{1E}=0$. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1E}\neq 0$, $C_{2E}\neq 0$, $C_{3E}=0$. With regard to Equations (2.33), (2.34), (5.4)–(5.11), (2.22), (2.23), we get

$$
\varepsilon_{nE}=-\left[\zeta_{1E}\left(\frac{1}{3}-\ln x_n\right)-c_{3E}\zeta_{2E}x_n^{c_{3E}-1}\right], \qquad (5.54)
$$

$$
\varepsilon_{\varphi E}=\varepsilon_{\theta E}=-\left[\zeta_{1E}\left(\frac{4}{3}-\ln x_n\right)+\zeta_{2E}x_n^{c_{3E}-1}\right] \qquad (5.55)
$$

$$
\varepsilon_{n\varphi E}=s_{44E}\,\sigma_{n\varphi E}=-\left[\left(\frac{4}{3}-\ln x_n\right)\frac{\partial\zeta_{1E}}{\partial\varphi}+x_n^{c_{3E}-1}\frac{\partial\zeta_{2E}}{\partial\varphi}\right], \qquad (5.56)
$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\,\Theta\left[\left(\frac{4}{3} - \ln x_n\right)\frac{\partial\zeta_{1E}}{\partial\nu} + x_n^{c_{3E}-1}\,\frac{\partial\zeta_{2E}}{\partial\nu}\right], \tag{5.57}$$

$$\begin{aligned}\sigma_{nE} = -\Bigg\{&\zeta_{1E}\left[\frac{c_{1E} - 7\,c_{2E}}{3} - (c_{1E} - c_{2E})\ln x_n\right] \\ &+ \zeta_{2E}\left[(c_{1E} + c_{2E})\,c_{3E} - 2\,c_{2E}\right]x_n^{c_{3E}-1}\Bigg\},\end{aligned} \tag{5.58}$$

$$\begin{aligned}\sigma_{\varphi E} = \sigma_{\theta E} = -\Bigg\{&\zeta_{1E}\left[\frac{4\,c_{1E} - c_{2E}}{3} - (c_{1E} - c_{2E})\ln x_n\right] \\ &+ \zeta_{2E}\,(c_{1E} - c_{2E}\,c_{3E})\,x_n^{c_{3E}-1}\Bigg\},\end{aligned} \tag{5.59}$$

$$\begin{aligned}w_E = {}&\kappa_{1E}\,\zeta_{1E}^2 + \kappa_{2E}\,\zeta_{2E}^2 + \kappa_{4E}\,\zeta_{1E}\,\zeta_{2E} + \frac{\chi_1}{s_{44E}}\left[\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{1E}}{\partial\nu}\right)^2\right] \\ &+ \frac{\chi_2}{s_{44E}}\left[\left(\frac{\partial\zeta_{2E}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2E}}{\partial\nu}\right)^2\right] + \frac{\chi_4}{s_{44E}}\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\,\frac{\partial\zeta_{2E}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{1E}}{\partial\nu}\,\frac{\partial\zeta_{2E}}{\partial\nu}\right),\end{aligned} \tag{5.60}$$

$$\begin{aligned}W_E = {}&4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{1E}\,\zeta_{1E}^2 + \Phi_{2E}\,\zeta_{2E}^2 + \Phi_4\,\zeta_{1E}\,\zeta_{2E}\right)\,d\varphi\,d\nu \\ &+ \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1E}\left[\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{1E}}{\partial\nu}\right)^2\right]\,d\varphi\,d\nu \\ &+ \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2E}\left[\left(\frac{\partial\zeta_{2E}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2E}}{\partial\nu}\right)^2\right]\,d\varphi\,d\nu \\ &+ \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{4E}\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\,\frac{\partial\zeta_{2E}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{1E}}{\partial\nu}\,\frac{\partial\zeta_{2E}}{\partial\nu}\right)\,d\varphi\,d\nu,\end{aligned} \tag{5.61}$$

where Θ, x_E, s_{44E}, c_{iE} $(i=1,2,3)$ and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{je} $(j=1,3,4)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.13); (4.15), respectively, and ζ_E, ζ_{iE} $(i=1,2)$ have the forms

$$\zeta_{iE} = p_{1n}\,\zeta_{i1E} + p_{2n}\,\zeta_{i2E}, \quad i = 1, 2,$$

$$
\begin{aligned}
\zeta_{11E} &= \frac{1}{\zeta_E}\left[(c_{1E}+c_{2E})\,c_{3E}-2\,c_{2E}\right]x_E^{c_{3E}-1},\\
\zeta_{12E} &= -\,\frac{1}{\zeta_E}\left[(c_{1E}+c_{2E})\,c_{3E}-2\,c_{2E}\right]x_{IN}^{c_{3E}-1},\\
\zeta_{21E} &= -\,\frac{1}{\zeta_E}\left[\frac{c_{1E}-7\,c_{2E}}{3}-(c_{1E}-c_{2E})\ln x_E\right],\\
\zeta_{22E} &= \frac{1}{\zeta_E}\left[\frac{c_{1E}-7\,c_{2E}}{3}-(c_{1E}-c_{2E})\ln x_{IN}\right],\\
\zeta_E &= \left[(c_{1E}+c_{2E})\,c_{3E}-2\,c_{2E}\right]\\
&\quad\times\left[\frac{c_{1E}-7\,c_{2E}}{3}\left(x_E^{c_{3E}-1}-x_{IN}^{c_{3E}-1}\right)\right.\\
&\qquad\left.-\;(c_{1E}-c_{2E})\left(x_E^{c_{3E}-1}\ln x_{IN}-x_{IN}^{c_{3E}-1}\ln x_E\right)\right],
\end{aligned}
\tag{5.62}
$$

The normal stresses p_{1n}, p_{2n} in Equation (5.62) are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (5.23), (5.33), (5.43), (5.53), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}}=-\left(p_{1n}\,\rho_{11E}+p_{2n}\,\rho_{12E}\right)$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E}=,-\left(p_{1n}\,\rho_{21E}+p_{2n}\,\rho_{22E}\right)$ and Equation (5.55), the coefficient ρ_{ijE} $(i,j=1,2)$ in Equations (2.25), (2.26) is derived as

$$
\rho_{1iE}=\zeta_{1iE}\left(\frac{4}{3}-\ln x_{IN}\right)+\zeta_{2iE}\,x_{IN}^{c_{3E}-1},\quad \rho_{2iE}=\zeta_{1iE}\left(\frac{4}{3}-\ln x_E\right)+\zeta_{2iE}\,x_E^{c_{3E}-1},\quad i=1,2.
\tag{5.63}
$$

Conditions $C_{1E}\neq 0$, $C_{3E}\neq 0$, $C_{2E}=0$. With regard to Equations (2.33), (2.34), (5.4)–(5.11), (2.22), (2.23), we get

$$
\varepsilon_{nE}=-\left[\zeta_{1E}\left(\frac{1}{3}-\ln x_n\right)+\frac{\zeta_{3E}}{x_n}\right],
\tag{5.64}
$$

$$
\varepsilon_{\varphi E}=\varepsilon_{\theta E}=-\left[\zeta_{1E}\left(\frac{4}{3}-\ln x_n\right)+\frac{\zeta_{3E}}{x_n}\left(\frac{1}{2}+\ln x_n\right)\right],
\tag{5.65}
$$

$$
\varepsilon_{n\varphi E}=s_{44E}\,\sigma_{n\varphi E}=-\left[\frac{\partial\zeta_{1E}}{\partial\varphi}\left(\frac{4}{3}-\ln x_n\right)+\frac{1}{x_n}\frac{\partial\zeta_{3E}}{\partial\varphi}\left(\frac{1}{2}+\ln x_n\right)\right],
\tag{5.66}
$$

$$
\varepsilon_{n\theta E}=s_{44E}\,\sigma_{n\theta E}=-\left[\frac{\partial\zeta_{1E}}{\partial\nu}\left(\frac{4}{3}-\ln x_n\right)+\frac{1}{x_n}\frac{\partial\zeta_{3E}}{\partial\nu}\left(\frac{1}{2}+\ln x_n\right)\right],
\tag{5.67}
$$

$$\sigma_{nE} = -\left\{\zeta_{1E}\left[\frac{2\,(c_{1E}+2\,c_{2E})}{3}+(c_{1E}-c_{2E})\ln x_n\right]-\frac{2\,c_{2E}\,\zeta_{3E}}{x_n}\right\}, \tag{5.68}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\left\{\zeta_{1E}\left[\frac{c_{1E}+2\,c_{2E}}{3}-(c_{1E}-c_{2E})\ln x_n\right]+\frac{c_{1E}\,\zeta_{3E}}{x_n}\right\}. \tag{5.69}$$

$$\begin{aligned} w_E &= \kappa_{1E}\,\zeta_{1E}^2+\kappa_{3E}\,\zeta_{3E}^2+\kappa_{5E}\,\zeta_{1E}\,\zeta_{3E}+\frac{\chi_{1E}}{s_{44E}}\left[\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{1E}}{\partial\nu}\right)^2\right] \\ &+\frac{\chi_{3E}}{s_{44E}}\left[\left(\frac{\partial\zeta_{3E}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{3E}}{\partial\nu}\right)^2\right]+\frac{\chi_{4E}}{s_{44E}}\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\,\frac{\partial\zeta_{3E}}{\partial\varphi}+\Theta^2\frac{\partial\zeta_{1E}}{\partial\nu}\,\frac{\partial\zeta_{3E}}{\partial\nu}\right), \end{aligned} \tag{5.70}$$

$$\begin{aligned} W_E &= 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{1E}\,\zeta_{1E}^2+\Phi_{3E}\,\zeta_{3E}^2+\Phi_5\,\zeta_{1E}\,\zeta_{3E}\right)\,d\varphi\,d\nu \\ &+\frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1E}\left[\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{1E}}{\partial\nu}\right)^2\right]\,d\varphi\,d\nu \\ &+\frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{3E}\left[\left(\frac{\partial\zeta_{3E}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{3E}}{\partial\nu}\right)^2\right]\,d\varphi\,d\nu \\ &+\frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{5E}\left(\frac{\partial\zeta_{1E}}{\partial\varphi}\,\frac{\partial\zeta_{3E}}{\partial\varphi}+\Theta^2\frac{\partial\zeta_{1E}}{\partial\nu}\,\frac{\partial\zeta_{3E}}{\partial\nu}\right)\,d\varphi\,d\nu, \end{aligned} \tag{5.71}$$

where Θ, x_E, s_{44E}, c_{iE} $(i=1,2,3)$ and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} $(j=1,3,5)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.13); (4.15), respectively, and ζ_{iE} $(i=1,3)$ has the forms

$$\begin{aligned} &\zeta_{iE} = p_{1n}\,\zeta_{i1E}+p_{2n}\,\zeta_{i2E}, \quad i=1,3, \\ &\zeta_{11E} = \frac{c_{1E}-2\,c_{2E}\ln x_E}{\zeta_E\,x_E}, \quad \zeta_{12E} = -\frac{c_{1E}-2\,c_{2E}\ln x_{IN}}{\zeta_E\,x_{IN}}, \\ &\zeta_{31E} = -\frac{1}{\zeta_E}\left[\frac{c_{1E}-7\,c_{2E}}{3}-(c_{1E}-c_{2E})\ln x_E\right], \end{aligned}$$

$$\zeta_{32E} = \frac{1}{\zeta_E}\left[\frac{c_{1E}-7c_{2E}}{3} - (c_{1E}-c_{2E})\ln x_{IN}\right],$$
$$\zeta_E = \frac{c_{1E}-7c_{2E}}{3}\left(\frac{c_{1E}-2c_{2E}\ln x_E}{x_E} - \frac{c_{1E}-2c_{2E}\ln x_{IN}}{x_{IN}}\right)$$
$$-\ (c_{1E}-c_{2E})\left[\frac{(c_{1E}-2c_{2E}\ln x_E)\ln x_{IN}}{x_E} - \frac{(c_{1E}-2c_{2E}\ln x_{IN})\ln x_E}{x_{IN}}\right]. \quad (5.72)$$

The normal stresses p_{1n}, p_{2n} in Equation (5.62) are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (5.23), (5.33), (5.43), (5.53), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}} = -(p_{1n}\rho_{11E}+p_{2n}\rho_{12E})$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E} = -(p_{1n}\rho_{21E}+p_{2n}\rho_{22E})$ and Equation (5.65), the coefficient ρ_{ijE} $(i,j=1{,}2)$ in Equations (2.25), (2.26) is derived as

$$\rho_{1iE} = \zeta_{1iE}\left(\frac{4}{3}-\ln x_{IN}\right) + \frac{\zeta_{3iE}}{x_{IN}}\left(\frac{1}{2}+\ln x_{IN}\right),$$
$$\rho_{2iE} = \zeta_{1iE}\left(\frac{4}{3}-\ln x_E\right) + \frac{\zeta_{3iE}}{x_E 2}\left(\frac{1}{2}+\ln x_E\right), \quad i=1,2. \quad (5.73)$$

Conditions $C_{2E} \neq 0$, $C_{3E} \neq 0$, $C_{1E} = 0$. With regard to Equations (2.33), (2.34), (5.4)–(5.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\left[c_{3E}\zeta_{2E}\, x_n^{c_{3E}-1} + \frac{\zeta_{3E}}{x_n}\right], \quad (5.74)$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\left[\zeta_{2E}\, x_n^{c_{3E}-1} + \frac{\zeta_{3E}}{x_n}\left(\frac{1}{2}+\ln r\right)\right], \quad (5.75)$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\left[\frac{\partial\zeta_{2E}}{\partial\varphi}x_n^{c_{3E}-1} + \frac{1}{x_n}\frac{\partial\zeta_{3E}}{\partial\varphi}\left(\frac{1}{2}+\ln r\right)\right], \quad (5.76)$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\left[\frac{\partial\zeta_{2E}}{\partial\nu}x_n^{c_{3E}-1} + \frac{1}{x_n}\frac{\partial\zeta_{3E}}{\partial\nu}\left(\frac{1}{2}+\ln r\right)\right], \quad (5.77)$$

$$\sigma_{nE} = -\left[\zeta_{2E}\,[(c_{1E}+c_{2E})\,c_{3E}-2c_{2E}]\,x_n^{c_{3E}-1} + \frac{\zeta_{3E}\,(c_{1E}-2c_{2E}\ln r)}{x_n}\right], \quad (5.78)$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\left[\zeta_{2E}\,(c_{1E}-c_{2E}\,c_{3E})\,x_n^{c_{3E}-1} + \frac{\zeta_{3E}}{x_n}\left(\frac{c_{1E}-2c_{2E}}{2}+c_{1E}\ln r\right)\right]. \quad (5.79)$$

$$w_E = \kappa_{2E}\,\zeta_{2E}^2 + \kappa_{3E}\,\zeta_{3E}^2 + \kappa_{6E}\,\zeta_{2E}\,\zeta_{3E} + \frac{\chi_{2E}}{s_{44E}}\left[\left(\frac{\partial\zeta_{2E}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2E}}{\partial\nu}\right)^2\right]$$
$$+ \frac{\chi_{3E}}{s_{44E}}\left[\left(\frac{\partial\zeta_{3E}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{3E}}{\partial\nu}\right)^2\right] + \frac{\chi_{6E}}{s_{44E}}\left(\frac{\partial\zeta_{2E}}{\partial\varphi}\frac{\partial\zeta_{3E}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{2E}}{\partial\nu}\frac{\partial\zeta_{3E}}{\partial\nu}\right), \tag{5.80}$$

$$W_E = 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{1E}\,\zeta_{2E}^2 + \Phi_{3E}\,\zeta_{3E}^2 + \Phi_5\,\zeta_{2E}\,\zeta_{3E}\right)\,d\varphi\,d\nu$$
$$+ \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1E}\left[\left(\frac{\partial\zeta_{2E}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2E}}{\partial\nu}\right)^2\right]\,d\varphi\,d\nu$$
$$+ \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{3E}\left[\left(\frac{\partial\zeta_{3E}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{3E}}{\partial\nu}\right)^2\right]\,d\varphi\,d\nu$$
$$+ \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{5E}\left(\frac{\partial\zeta_{2E}}{\partial\varphi}\frac{\partial\zeta_{3E}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{2E}}{\partial\nu}\frac{\partial\zeta_{3E}}{\partial\nu}\right)\,d\varphi\,d\nu, \tag{5.81}$$

where Θ, x_E, s_{44E}, c_{iE} $(i=1,2,3)$ and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} $(j=2,3,6)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (4.13); (4.15), respectively, and ζ_{iE} $(i=2,3)$ has the forms

$$\begin{aligned}
&\zeta_{iE} = p_{1n}\,\zeta_{i1E} + p_{2n}\,\zeta_{i2E}, \quad i = 2,3,\\
&\zeta_{21} = \frac{c_{1E} - 2\,c_{2E}\ln x_E}{\zeta_E\,x_E}, \quad \zeta_{22E} = -\frac{c_{1E} - 2\,c_{2E}\ln x_{IN}}{\zeta_E\,x_{IN}},\\
&\zeta_{31E} = -\frac{\left[(c_{1E} + c_{2E})\,c_{3E} - 2\,c_{2E}\right]x_E^{c_{3E}-1}}{\zeta_E},\\
&\zeta_{32E} = \frac{\left[(c_{1E} + c_{2E})\,c_{3E} - 2\,c_{2E}\right]x_{IN}^{c_{3E}-1}}{\zeta_E},\\
&\zeta_E = \left[(c_{1E} + c_{2E})\,c_{3E} - 2\,c_{2E}\right]\\
&\qquad\times\left[\frac{x_{IN}^{c_{3E}-1}\,(c_{1E} - 2\,c_{2E}\ln x_E)}{x_E} - \frac{x_E^{c_{3E}-1}\,(c_{1E} - 2\,c_{2e}\ln x_{IN})}{x_{IN}}\right].
\end{aligned} \tag{5.82}$$

The normal stresses p_{1n}, p_{2n} in Equation (5.62) are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (5.23), (5.33), (5.43), (5.53), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}} = -\left(p_{1n}\rho_{11E}+p_{2n}\rho_{12E}\right)$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E} = -\left(p_{1n}\rho_{21E}+p_{2n}\rho_{22E}\right)$ and Equation (5.75), the coefficient ρ_{ijE} $(i,j=1,2)$ in Equations (2.25), (2.26) is derived as

$$\rho_{1iE} = \zeta_{2iE}\,x_{IN}^{c_{3E}-1} + \frac{\zeta_{3iE}}{x_{IN}}\left(\frac{1}{2}+\ln x_{IN}\right), \quad \rho_{2iE} = \zeta_{2iE}\,x_{E}^{c_{3E}-1} + \frac{\zeta_{3iE}}{x_{E}}\left(\frac{1}{2}+\ln x_{E}\right),$$
$$i=1,2. \tag{5.83}$$

5.3 Condition $\beta_{IN} \neq \beta_E = \beta_M$

If $\beta_{IN} \neq \beta_E = \beta_M$, then $|u_{nE}| = f(x_n)$ represents a decreasing function of $x_n \in \langle x_{IN}, x_E\rangle$, and then $C_{1E} = C_{3E} = 0$, $C_{2E} \neq 0$ (see Equation (5.4)), respectively. With regard to Equations (4.4), (5.4), the thermal strains and stresses in the ellipsoidal envelope are determined in Section 4.3.2. The normal stress p_{1n} in Section 4.3.2 is given by Equation (2.27), where ρ_{IN} and ρ_{1ME} in Equation (2.27) are given by Equations (6.89) and (4.130), respectively.

5.3.1 Matrix

The integration constants C_{1M}, C_{2M}, C_{3M} for the matrix (see Equation (5.4)) are determined by the boundary conditions (2.30), (2.31) or (2.30)–(2.32). The boundary conditions result in the following combinations of C_{1M}, C_{2M}, C_{3M}, i.e., $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$; $C_{1M} \neq 0$, $C_{3M} \neq 0$, $C_{2M} = 0$; $C_{2M} \neq 0$, $C_{3M} \neq 0$, $C_{1M} = 0$; $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} \neq 0$. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$. With regard to Equations (2.30), (2.31), (5.4)–(5.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = \zeta_{1M}\left(\frac{1}{3}-\ln x_n\right) + c_{3M}\,\zeta_{2M}\,x_n^{c_{3M}-1}, \tag{5.84}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = \zeta_{1M}\left(\frac{4}{3}-\ln x_n\right) + \zeta_{2M}\,x_n^{c_{3M}-1}, \tag{5.85}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = \left(\frac{4}{3}-\ln x_n\right)\frac{\partial\zeta_{1M}}{\partial\varphi} + x_n^{c_{3M}-1}\frac{\partial\zeta_{2M}}{\partial\varphi}, \tag{5.86}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = \Theta\left[\left(\frac{4}{3}-\ln x_n\right)\frac{\partial\zeta_{1M}}{\partial\nu} + x_n^{c_{3M}-1}\frac{\partial\zeta_{2M}}{\partial\nu},\right], \tag{5.87}$$

$$\sigma_{nM} = -\,\zeta_{1M}\left[\frac{c_{1M} - 7\,c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right] + \zeta_{2M}\left[(c_{1M} + c_{2M})\,c_{3M} - 2\,c_{2M}\right] x_n^{c_{3M}-1}, \tag{5.88}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = \zeta_{1M}\left[\frac{4c_{1M} - c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right] + \zeta_{2M}\,(c_{1M} - c_{2M}\,c_{3M})\,x_n^{c_{3M}-1}, \tag{5.89}$$

$$\begin{aligned} w_M = {} & \kappa_{1M}\,\zeta_{1M}^2 + \kappa_{2M}\,\zeta_{2M}^2 + \kappa_{4M}\,\zeta_{1M}\,\zeta_{2M} + \frac{\chi_{1M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{1M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{1M}}{\partial\nu}\right)^2\right] \\ & + \frac{\chi_{2M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{2M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2M}}{\partial\nu}\right)^2\right] + \frac{\chi_{4M}}{s_{44M}}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\,\frac{\partial\zeta_{2M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\,\frac{\partial\zeta_{2M}}{\partial\nu}\right], \end{aligned} \tag{5.90}$$

$$\begin{aligned} W_M = {} & 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{1M}\,\zeta_{1M}^2 + \Phi_{2M}\,\zeta_{2M}^2 + \Phi_{4M}\,\zeta_{1M}\,\zeta_{2M}\right) d\varphi\, d\nu \\ & + \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1M}\left[\left(\frac{\partial\zeta_{1M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{1M}}{\partial\nu}\right)^2\right] d\varphi\, d\nu \\ & + \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2M}\left[\left(\frac{\partial\zeta_{2M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2M}}{\partial\nu}\right)^2\right] d\varphi\, d\nu \\ & + \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{4M}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\,\frac{\partial\zeta_{2M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\,\frac{\partial\zeta_{2M}}{\partial\nu}\right] d\varphi\, d\nu, \end{aligned} \tag{5.91}$$

where Θ, x_E, x_M, s_{44M}, c_{iM} and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} $(i=1,2,3;\ j=1,2,4)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (5.12); (5.13), respectively, and ζ_{1M}, ζ_{2M}, ζ_M have the forms

$$\begin{aligned} & \zeta_{1M} = -\frac{p_{1n}\,\rho_{2ME}\,x_E\,x_M^{c_{3M}}}{\zeta_M}, \quad \zeta_{2M} = \frac{p_{1n}\,\rho_{2ME}\,x_E\,x_M}{\zeta_M}\left(\frac{4}{3} - \ln x_M\right), \\ & \zeta_M = x_M^{c_{3M}}\,x_E\left(\frac{4}{3} - \ln x_E\right) - x_M\,x_E^{c_{3M}}\left(\frac{4}{3} - \ln x_M\right). \end{aligned} \tag{5.92}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{3M} \neq 0$, $C_{2M} = 0$. With regard to Equations (2.30), (2.31), (5.4)–(5.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = \zeta_{1M}\left(\frac{1}{3} - \ln x_n\right) + \frac{\zeta_{3M}}{x_n}, \tag{5.93}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = \zeta_{1M}\left(\frac{4}{3} - \ln x_n\right) + \frac{\zeta_{3M}}{x_n}\left(\frac{1}{2} + \ln x_n\right), \tag{5.94}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = \left(\frac{4}{3} - \ln x_n\right)\frac{\partial \zeta_{1M}}{\partial \varphi} + \frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial \zeta_{3M}}{\partial \varphi}, \tag{5.95}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = \Theta\left[\left(\frac{4}{3} - \ln x_n\right)\frac{\partial \zeta_{1M}}{\partial \nu} + \frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial \zeta_{3M}}{\partial \nu}\right], \tag{5.96}$$

$$\sigma_{nM} = \zeta_{1M}\left[\frac{c_{1M} - 7\,c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right] + \frac{\zeta_{3M}\,(c_{1M} - 2\,c_{2M}\ln x_n)}{x_n}, \tag{5.97}$$

$$\begin{aligned}\sigma_{\varphi M} = \sigma_{\theta M} = {}& \zeta_{1M}\left[\frac{4\,c_{1M} - c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right] \\ & + \frac{\zeta_{3M}}{x_n}\left(\frac{c_{1M} - 2\,c_{2M}}{2} + c_{1M}\ln x_n\right),\end{aligned} \tag{5.98}$$

$$\begin{aligned}w_M = {}& \kappa_{1M}\,\zeta_{1M}^2 + \kappa_{3M}\,\zeta_{3M}^2 + \kappa_{5M}\,\zeta_{1M}\,\zeta_{3M} + \frac{\chi_{1M}}{s_{44M}}\left[\left(\frac{\partial \zeta_{1M}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{1M}}{\partial \nu}\right)^2\right] \\ & + \frac{\chi_{3M}}{s_{44M}}\left[\left(\frac{\partial \zeta_{3M}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{3M}}{\partial \nu}\right)^2\right] + \frac{\chi_{5M}}{s_{44M}}\left[\frac{\partial \zeta_{1M}}{\partial \varphi}\frac{\partial \zeta_{3M}}{\partial \varphi} + \Theta^2\frac{\partial \zeta_{1M}}{\partial \nu}\frac{\partial \zeta_{3M}}{\partial \nu}\right],\end{aligned} \tag{5.99}$$

$$W_M = 4\int 0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{1M}\,\zeta_{1M}^2 + \Phi_{3M}\,\zeta_{3M}^2 + \Phi_{5M}\,\zeta_{1M}\,\zeta_{3M}\right)d\varphi\,d\nu$$

$$+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1M}\left[\left(\frac{\partial\zeta_{1M}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{1M}}{\partial\nu}\right)^2\right]d\varphi\,d\nu$$

$$+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{3M}\left[\left(\frac{\partial\zeta_{3M}}{\partial\varphi}\right)^2+\Theta^2\left(\frac{\partial\zeta_{3M}}{\partial\nu}\right)^2\right]d\varphi\,d\nu$$

$$+\frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{5M}\left[\frac{\partial\zeta_{1M}}{\partial\varphi}\frac{\partial\zeta_{3M}}{\partial\varphi}+\Theta^2\frac{\partial\zeta_{1M}}{\partial\nu}\frac{\partial\zeta_{3M}}{\partial\nu}\right]d\varphi\,d\nu, \qquad (5.100)$$

where Θ, x_E, x_M, s_{44M}, c_{iM} and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} ($i=1,2,3;\ j=1,3,5$) are given by Equations (1.21), (1.22), (2.13), (2.18) and (5.12); (5.13), respectively, and ζ_{1M}, ζ_{3M}, ζ_M have the forms

$$\zeta_{1M}=-\frac{p_{1n}\,\rho_{2ME}\,x_E}{\zeta_M}\left(\frac{1}{2}+\ln x_M\right),\quad \zeta_{3M}=\frac{p_{1n}\,\rho_{2ME}\,x_E\,x_M}{\zeta_M}\left(\frac{4}{3}-\ln x_M\right),$$

$$\zeta_M=x_E\left(\frac{4}{3}-\ln x_E\right)\left(\frac{1}{2}+\ln x_M\right)-x_M\left(\frac{4}{3}-\ln x_M\right)\left(\frac{1}{2}+\ln x_E\right). \qquad (5.101)$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{2M}\neq 0$, $C_{3M}\neq 0$, $C_{1M}=0$. With regard to Equations (2.30), (2.31), (5.4)–(5.11), (2.22), (2.23), we get

$$\varepsilon_{nM}=c_{3M}\,\zeta_{2M}\,x_n^{c_{3M}-1}+\frac{\zeta_{3M}}{x_n}, \qquad (5.102)$$

$$\varepsilon_{\varphi M}=\varepsilon_{\theta M}=\zeta_{2M}\,x_n^{c_{3M}-1}+\frac{\zeta_{3M}}{x_n}\left(\frac{1}{2}+\ln x_n\right), \qquad (5.103)$$

$$\varepsilon_{n\varphi M}=s_{44M}\,\sigma_{n\varphi M}=x_n^{c_{3M}-1}\frac{\partial\zeta_{2M}}{\partial\varphi}+\frac{1}{x_n}\left(\frac{1}{2}+\ln x_n\right)\frac{\partial\zeta_{3M}}{\partial\varphi}, \qquad (5.104)$$

$$\varepsilon_{n\theta M}=s_{44M}\,\sigma_{n\theta M}=\Theta\left[x_n^{c_{3M}-1}\frac{\partial\zeta_{2M}}{\partial\nu}+\frac{1}{x_n}\left(\frac{1}{2}+\ln x_n\right)\frac{\partial\zeta_{3M}}{\partial\nu}\right], \qquad (5.105)$$

$$\sigma_{nM}=\zeta_{2M}\left[(c_{1M}+c_{2M})\,c_{3M}-2\,c_{Mm}\right]x_n^{c_{3M}-1}+\frac{\zeta_{3M}}{x_n}\left(c_{1M}-2\,c_{2M}\ln x_n\right), \qquad (5.106)$$

$$\sigma_{\varphi M} = \sigma'_{\theta M} = \zeta_{2M}\left(c_{1M} - c_{2M}\,c_{3M}\right)x_n^{c_{3M}-1} + \frac{\zeta_{3M}}{x_n}\left(\frac{c_{1M} - 2\,c_{2M}}{2} + c_{1M}\ln x_n\right), \tag{5.107}$$

$$\begin{aligned} w_M &= \kappa_{2M}\,\zeta_{2M}^2 + \kappa_{3M}\,\zeta_{3M}^2 + \kappa_{6M}\,\zeta_{2M}\,\zeta_{3M} + \frac{\chi_{2M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{2M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2M}}{\partial\nu}\right)^2\right] \\ &+ \frac{\chi_{3M}}{s_{44M}}\left[\left(\frac{\partial\zeta_{3M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{3M}}{\partial\nu}\right)^2\right] + \frac{\chi_{6M}}{s_{44M}}\left[\frac{\partial\zeta_{2M}}{\partial\varphi}\,\frac{\partial\zeta_{3M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{2M}}{\partial\nu}\,\frac{\partial\zeta_{3M}}{\partial\nu}\right], \end{aligned} \tag{5.108}$$

$$\begin{aligned} W_M &= 4\int_0^{\pi/2}\int_0^{\pi/2}\left(\Phi_{2M}\,\zeta_{2M}^2 + \Phi_{3M}\,\zeta_{3M}^2 + \Phi_{6M}\,\zeta_{2M}\,\zeta_{3M}\right)d\varphi\,d\nu \\ &+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{2M}\left[\left(\frac{\partial\zeta_{2M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{2M}}{\partial\nu}\right)^2\right]d\varphi\,d\nu \\ &+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{3M}\left[\left(\frac{\partial\zeta_{3M}}{\partial\varphi}\right)^2 + \Theta^2\left(\frac{\partial\zeta_{3M}}{\partial\nu}\right)^2\right]d\varphi\,d\nu \\ &+ \frac{4}{s_{44M}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{6M}\left[\frac{\partial\zeta_{2M}}{\partial\varphi}\,\frac{\partial\zeta_{3M}}{\partial\varphi} + \Theta^2\frac{\partial\zeta_{2M}}{\partial\nu}\,\frac{\partial\zeta_{3M}}{\partial\nu}\right]d\varphi\,d\nu, \end{aligned} \tag{5.109}$$

where Θ, x_E, x_M, s_{44M}, c_{iM} and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} $(i=1,2,3;\ j=2,3,6)$ are given by Equations (1.21), (1.22), (2.13), (2.18) and (5.12); (5.13), respectively, and ζ_{2M}, ζ_{3M}, ζ_M have the forms

$$\begin{aligned} &\zeta_{2M} = -\frac{p_{1n}\,\rho_{2ME}\,x_E}{\zeta_M}\left(\frac{1}{2} + \ln x_M\right), \quad \zeta_{3M} = \frac{p_{1n}\,\rho_{2ME}\,x_E\,x_M^{c_{3M}}}{\zeta_M}, \\ &\zeta_M = x_E^{c_{3M}}\left(\frac{1}{2} + \ln x_M\right) - x_M^{c_{3M}}\left(\frac{1}{2} + \ln x_E\right). \end{aligned} \tag{5.110}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} \neq 0$. With regard to Equations (2.30), (2.31), (2.32), (5.4)–(5.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = \zeta_{1M}\left(\frac{1}{3} - \ln x_n\right) + c_{3M}\,\zeta_{2M}\,x_n^{c_{3M}-1} + \frac{\zeta_{3M}}{x_n}, \tag{5.111}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = \zeta_{1M}\left(\frac{4}{3} - \ln x_n\right) + \zeta_{2M}\,x_n^{c_{3M}-1} + \frac{\zeta_{3M}}{x_n}\left(\frac{1}{2} + \ln x_n\right), \tag{5.112}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = \left(\frac{4}{3} - \ln x_n\right)\frac{\partial \zeta_{1M}}{\partial \varphi} + x_n^{c_{3M}-1}\frac{\partial \zeta_{2M}}{\partial \varphi} + \frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial \zeta_{3M}}{\partial \varphi}, \tag{5.113}$$

$$\begin{aligned}\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = &\\ \Theta\left[\left(\frac{4}{3} - \ln x_n\right)\frac{\partial \zeta_{1M}}{\partial \nu} + x_n^{c_{3M}-1}\frac{\partial \zeta_{2M}}{\partial \nu} + \frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial \zeta_{3M}}{\partial \nu}\right],\end{aligned} \tag{5.114}$$

$$\begin{aligned}\sigma_{nM} = {} & \zeta_{1M}\left[\frac{c_{1M} - 7\,c_{2M}}{3} - (c_{1M} - c_{Mm})\ln x_n\right] \\ & + \zeta_{2M}\left[(c_{1M} + c_{2M})\,c_{3M} - 2\,c_{2M}\right]x_n^{c_{3M}-1} + \frac{\zeta_{3M}}{x_n}\left(c_{1M} - 2\,c_{2M}\ln x_n\right),\end{aligned} \tag{5.115}$$

$$\begin{aligned}\sigma_{\varphi M} = \sigma_{\theta M} = {} & \zeta_{1M}\left[\frac{4\,c_{1M} - c_{2M}}{3} - (c_{1M} - c_{2M})\ln x_n\right] \\ & + \zeta_{2M}\left(c_{1M} - c_{2M}\,c_{3M}\right)x_n^{c_{Mm}-1} + \frac{\zeta_{3M}}{x_n}\left(\frac{c_{1M} - 2\,c_{2M}}{2} + c_{1M}\ln x_n\right),\end{aligned} \tag{5.116}$$

$$\begin{aligned}w_M = {} & \kappa_{1M}\,\zeta_{1M}^2 + \kappa_{2M}\,\zeta_{2M}^2 + \kappa_{3M}\,\zeta_{3M}^2 + \kappa_{4M}\,\zeta_{1M}\,\zeta_{2M} + \kappa_{5M}\,\zeta_{1M}\,\zeta_{3M} + \kappa_{6M}\,\zeta_{2M}\,\zeta_{3M} \\ & + \frac{\chi_{1M}}{s_{44M}}\left[\left(\frac{\partial \zeta_{1M}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{1M}}{\partial \nu}\right)^2\right] + \frac{\chi_{2M}}{s_{44M}}\left[\left(\frac{\partial \zeta_{2M}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{2M}}{\partial \nu}\right)^2\right] \\ & + \frac{\chi_{3M}}{s_{44M}}\left[\left(\frac{\partial \zeta_{3M}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial \zeta_{3M}}{\partial \nu}\right)^2\right] + \frac{\chi_{4M}}{s_{44M}}\left[\frac{\partial \zeta_{1M}}{\partial \varphi}\frac{\partial \zeta_{2M}}{\partial \varphi} + \Theta^2\frac{\partial \zeta_{1M}}{\partial \nu}\frac{\partial \zeta_{2M}}{\partial \nu}\right] \\ & + \frac{\chi_{5M}}{s_{44M}}\left[\frac{\partial \zeta_{1M}}{\partial \varphi}\frac{\partial \zeta_{3M}}{\partial \varphi} + \Theta^2\frac{\partial \zeta_{1M}}{\partial \nu}\frac{\partial \zeta_{3M}}{\partial \nu}\right] \\ & + \frac{\chi_{6M}}{s_{44M}}\left[\frac{\partial \zeta_{2M}}{\partial \varphi}\frac{\partial \zeta_{3M}}{\partial \varphi} + \Theta^2\frac{\partial \zeta_{2M}}{\partial \nu}\frac{\partial \zeta_{3M}}{\partial \nu}\right],\end{aligned} \tag{5.117}$$

$$
\begin{aligned}
W_M = {} & 4 \int_0^{\pi/2} \int_0^{\pi/2} \left(\Phi_{1M}\,\zeta_{1M}^2 + \Phi_{2M}\,\zeta_{2M}^2 + \Phi_{3M}\,\zeta_{3M}^2 \right) d\varphi\, d\nu \\
& + 4\text{¢} \int_0^{\pi/2} \int_0^{\pi/2} \left(\Phi_{4M}\,\zeta_{1M}\,\zeta_{2M} + \Phi_{5M}\,\zeta_{1M}\,\zeta_{3M} + \Phi_{6M}\,\zeta_{2M}\,\zeta_{3M} \right) d\varphi\, d\nu \\
& + \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{1M} \left[\left(\frac{\partial \zeta_{1M}}{\partial \varphi} \right)^2 + \Theta^2 \left(\frac{\partial \zeta_{1M}}{\partial \nu} \right)^2 \right] d\varphi\, d\nu \\
& + \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{2M} \left[\left(\frac{\partial \zeta_{2M}}{\partial \varphi} \right)^2 + \Theta^2 \left(\frac{\partial \zeta_{2M}}{\partial \nu} \right)^2 \right] d\varphi\, d\nu \\
& + \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{3M} \left[\left(\frac{\partial \zeta_{3M}}{\partial \varphi} \right)^2 + \Theta^2 \left(\frac{\partial \zeta_{3M}}{\partial \nu} \right)^2 \right] d\varphi\, d\nu \\
& + \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{4M} \left[\frac{\partial \zeta_{1M}}{\partial \varphi} \frac{\partial \zeta_{2M}}{\partial \varphi} + \Theta^2 \frac{\partial \zeta_{1M}}{\partial \nu} \frac{\partial \zeta_{2M}}{\partial \nu} \right] d\varphi\, d\nu \\
& + \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{5M} \left[\frac{\partial \zeta_{1M}}{\partial \varphi} \frac{\partial \zeta_{3M}}{\partial \varphi} + \Theta^2 \frac{\partial \zeta_{1M}}{\partial \nu} \frac{\partial \zeta_{3M}}{\partial \nu} \right] d\varphi\, d\nu \\
& + \frac{4}{s_{44M}} \int_0^{\pi/2} \int_0^{\pi/2} \Psi_{6M} \left[\frac{\partial \zeta_{2M}}{\partial \varphi} \frac{\partial \zeta_{3M}}{\partial \varphi} + \Theta^2 \frac{\partial \zeta_{2M}}{\partial \nu} \frac{\partial \zeta_{3M}}{\partial \nu} \right] d\varphi\, d\nu,
\end{aligned}
\tag{5.118}
$$

where Θ, x_E, x_M, s_{44M}, c_{iM} and κ_{jE}, χ_{jE}; Φ_{jE}, Ψ_{jE} ($i=1,2,3;\ j=1,\ldots,6$) are given by Equations (1.21), (1.22), (2.13), (2.18) and (5.12); (5.13), respectively, and ζ_{iM} ($i=1,2,3$), ζ_M have the forms

$$
\begin{aligned}
\zeta_{1M} &= -\frac{p_{1n}\,\rho_{2ME}\,x_E\,x_M^{c_{3M}-1}}{\zeta_M} \left[1 - c_{3M} \left(\frac{1}{2} + \ln x_M \right) \right], \\
\zeta_{2M} &= \frac{p_{1n}\,\rho_{2ME}\,x_e}{\zeta_M} \left[\frac{4}{3} - \ln x_M - \left(\frac{1}{3} - \ln x_M \right) \left(\frac{1}{2} + \ln x_M \right) \right], \\
\zeta_{3M} &= -\frac{p_1\,\rho_{2ME}\,x_E\,x_M^{c_{3M}}}{\zeta_M} \left[c_{3M} \left(\frac{4}{3} - \ln x_M \right) - \left(\frac{1}{3} - \ln x_M \right) \right],
\end{aligned}
$$

$$\zeta_M = c_{3M} x_M^{c_{3M}-1}\left[x_E\left(\frac{1}{3}-\ln x_E\right)-x_M\left(\frac{1}{3}-\ln x_M\right)\right]$$
$$+\ (x_E^{c_{3M}} - x_M^{c_{3M}})\left(\frac{2}{3}+\ln x_M\right),$$
$$\zeta_M = x_M^{c_{3M}-1} x_E\left(\frac{4}{3}-\ln x_E\right)\left[1-c_{3M}\left(\frac{1}{2}+\ln x_M\right)\right]$$
$$-\ x_E^{c_{3M}}\left[\frac{4}{3}-\ \ln x_M-\left(\frac{1}{3}-\ln x_M\right)\left(\frac{1}{2}+\ln x_M\right)\right]$$
$$+\ x_M^{c_{3M}}\left(\frac{1}{2}+\ln x_E\right)\left[c_{3M}\left(\frac{4}{3}-\ln x_M\right)-\left(\frac{1}{3}-\ln x_M\right)\right]. \tag{5.119}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

5.4 Condition $\beta_{IN} = \beta_E \neq \beta_M$

If $\beta_{IN} = \beta_E \neq \beta_M$, then the thermal strains and stresses in the cell matrix are determined in Section 5.2.1. The radial stress p_{2n} in Section 5.2.1 is given by Equations (2.28). The coefficients ρ_M and ρ_{2INE} in Equations (2.26) are given by Equations (5.24), (5.34), (5.44), (5.54) and (5.129), (5.139), respectively.

5.4.1 Envelope

If $\beta_{IN} = \beta_E \neq \beta_M$, then $|u_{nE}| = f(x_n)$ represents an increasing function of $x_n \in \langle x_{IN}, x_E\rangle$, and then $C_{1E} \neq 0$, $C_{2E} = C_{3E} = 0$; or $C_{3E} \neq 0$, $C_{1E} = C_{3E} = 0$ (see Equation (5.4)).

Conditions $C_{1E} \neq 0$, $C_{2E} = C_{3E} = 0$. With regard to Equations (2.34), (5.4)–(5.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\ \frac{p_{2n}}{\zeta_E}\left(\frac{1}{3}-\ln x_n\right), \tag{5.120}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\ \frac{p_{2n}}{\zeta_E}\left(\frac{4}{3}-\ln x_n\right), \tag{5.121}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\ \left(\frac{4}{3}-\ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_E}\right), \tag{5.122}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\Theta\,\left(\frac{4}{3} - \ln x_n\right)\frac{\partial}{\partial \nu}\left(\frac{p_{2n}}{\zeta_E}\right), \tag{5.123}$$

$$\sigma_{nE} = -\frac{p_{2n}}{\zeta_E}\left[\frac{c_{1E} - 7\,c_{2E}}{3} - (c_{1E} - c_{2E})\ln x_n\right], \tag{5.124}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\frac{p_{2n}}{\zeta_E}\left[\frac{4\,c_{1E} - c_{2E}}{3} - (c_{1E} - c_{2E})\ln x_n\right], \tag{5.125}$$

$$w_E = \kappa_{1E}\left(\frac{p_{2n}}{\zeta_E}\right)^2 + \frac{\chi_{1E}}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2\right\}, \tag{5.126}$$

$$W_E = 4\int_0^{\pi/2}\int_0^{\pi/2}\Phi_{1E}\left(\frac{p_{2n}}{\zeta_E}\right)^2 d\varphi\, d\nu + \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{1E}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2\right\} d\varphi\, d\nu, \tag{5.127}$$

where Θ, x_E, s_{44E}, c_{iE} ($i = 1,2,3$) and κ_{1E}, χ_{1E}; Φ_{1E}, Ψ_{1E} are given by Equations (1.21), (1.22), (2.13), (2.18) and (5.12); (5.13), respectively, and ζ_E has the form

$$\zeta_E = \frac{c_{1E} - 7c_{2E}}{3} - (c_{1E} - c_{2E})\ln x_E. \tag{5.128}$$

The normal stress p_{2n} is given by Equation (2.28), where ρ_M in Equation (2.28) is given by Equations (3.24), (4.24), (4.34), (4.44), (4.54), (5.23), (5.33), (5.43), (5.53), (6.22), (6.32), (6.42), (6.52), (7.23), (7.33), (7.43), (7.53), (7.63), (7.73) with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n = x_{IN}} = -p_{2n}\,\rho_{1INE}$, $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E} = -p_{2n}\,\rho_{2INE}$ (see Equation (5.121)), the coefficients ρ_{1INE}, ρ_{2INE} in Equations (2.28), (2.37) are derived as

$$\rho_{1INE} = \frac{1}{\zeta_E}\left(\frac{4}{3} - \ln x_{IN}\right), \quad \rho_{2INE} = \frac{1}{\zeta_E}\left(\frac{4}{3} - \ln x_E\right). \tag{5.129}$$

Conditions $C_{3E} \neq 0, C_{1E} = C_{3E} = 0$. With regard to Equations (2.34), (5.4)–(5.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\frac{p_{2n}}{\zeta_E\, x_n}, \tag{5.130}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\frac{p_{2n}}{\zeta_E\, x_n}\left(\frac{1}{2} + \ln x_n\right), \tag{5.131}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\frac{1}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_E}\right), \tag{5.132}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\frac{\Theta}{x_n}\left(\frac{1}{2} + \ln x_n\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_E}\right), \tag{5.133}$$

$$\sigma_{nE} = -\frac{p_{2n}}{\zeta_E\, x_n}\left(c_{1E} - 2\,c_{2E}\ln x_n\right), \tag{5.134}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\frac{p_{2n}}{\zeta_E\, x_n}\left(\frac{c_{1E} - 2\,c_{2E}}{2} + c_{1E}\ln x_n\right), \tag{5.135}$$

$$w_E = \kappa_{3E}\left(\frac{p_{2n}}{\zeta_E}\right)^2 + \frac{\chi_{3E}}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2\right\}, \tag{5.136}$$

$$\begin{aligned} W_E = {} & 4\int_0^{\pi/2}\int_0^{\pi/2}\Phi_{3E}\left(\frac{p_{2n}}{\zeta_E}\right)^2 d\varphi\, d\nu \\ & + \frac{4}{s_{44E}}\int_0^{\pi/2}\int_0^{\pi/2}\Psi_{3E}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_E}\right)\right]^2\right\} d\varphi\, d\nu, \end{aligned} \tag{5.137}$$

where Θ, x_E, s_{44E}, c_{iE} $(i = 1,2,3)$ and κ_{3E}, χ_{3E}; Φ_{3E}, Ψ_{3E} are given by Equations (1.21), (1.22), (2.13), (2.18) and (5.12); (5.13), respectively, and ζ_E has the form

$$\zeta_E = \frac{c_{1E} - 2\,c_{2E}\ln x_E}{x_E}. \tag{5.138}$$

The normal stress p_{2n} is given by Equation (2.28), where ρ_M in Equation (2.28) is given by Equations (3.24), (4.24), (4.34), (4.44), (4.54), (5.23), (5.33), (5.43), (5.53), (6.22), (6.32), (6.42), (6.52), (7.23), (7.33), (7.43), (7.53), (7.63), (7.73) with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n = x_{IN}} = -p_{2n}\,\rho_{1INE}$, $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E} = -p_{2n}\,\rho_{2INE}$ (see Equation (5.121)), the coefficients ρ_{1INE}, ρ_{2INE} in Equations (2.28), (2.37) are derived as

$$\rho_{1INE} = \frac{1}{\zeta_E\, x_{IN}}\left(\frac{1}{2} + \ln x_{IN}\right), \quad \rho_{2INE} = \frac{1}{\zeta_E\, x_E}\left(\frac{1}{2} + \ln x_E\right). \tag{5.139}$$

Chapter 6

Mathematical Model 4

6.1 Mathematical Procedure

The differential equation (2.19) is transformed to the form

$$U_n = -s_{44}\left(c_1 + c_2\right)\left(x_n^2 \frac{\partial^2 u_n}{\partial x_n^2} + 2x_n \frac{\partial u_n}{\partial x_n} - 2u_n\right), \tag{6.1}$$

where s_{44}, c_i ($i = 1{,}2$) and $U_n = U_n\left(x_n, \varphi, \nu\right)$ are given by Equations (2.13), (2.18) and (2.21), respectively. Let $x_n\left[\partial \mathrm{Eq.}(6.1)/\partial x_n\right]$ be performed, and then we get

$$x_n \frac{\partial U_n}{\partial x_n} = -s_{44}\left(c_1 + c_2\right)\left(x_n^3 \frac{\partial^3 u_n}{\partial x_n^3} + 4x_n^2 \frac{\partial^2 u_n}{\partial x_n^2}\right). \tag{6.2}$$

Let Equations (6.1), (6.2 be substituted to Equation (2.20, and then we get

$$x_n^3 \frac{\partial^3 u_n}{\partial x_n^3} + \left(4 - c_3\right) x_n^2 \frac{\partial^2 u_n}{\partial x_n^2} - 2c_3 x_n \frac{\partial u_n}{\partial x_n} + 2c_3 u_n = 0. \tag{6.3}$$

Let u_n be assumed in the form $u_n = x_n^{\lambda}$, then we get [2]–[23]

$$u_n = C_1 x_n + C_2 x_n^{c_3} + \frac{C_3}{x_n^2}, \tag{6.4}$$

where $c_3 < 0$ is given by Equation (2.18), and C_1, C_2, C_3 are integration constants, which are determined by the boundary conditions in Section 2.4. With regard to Equations (2.1)–(2.4), (2.14)–(2.17), (2.22), (6.6), we get

$$\varepsilon_n = C_1 + C_2 c_3 x_n^{c_3 - 1} - \frac{2C_3}{x_n^3}, \tag{6.5}$$

$$\varepsilon_\varphi = \varepsilon_\theta = C_1 + C_2 x_n^{c_3 - 1} + \frac{C_3}{x_n^3}, \tag{6.6}$$

$$\varepsilon_{n\varphi} = s_{44}\,\sigma_{n\varphi} = \frac{\partial C_1}{\partial \varphi} + x_n^{c_3 - 1}\frac{\partial C_2}{\partial \varphi} + \frac{1}{x_n^3}\frac{\partial C_3}{\partial \varphi}, \tag{6.7}$$

$$\varepsilon_{n\theta} = s_{44}\,\sigma_{n\theta} = \Theta\left[\frac{\partial C_1}{\partial \nu} + x_n^{c_3 - 1}\frac{\partial C_2}{\partial \nu} + \frac{1}{x_n^3}\frac{\partial C_3}{\partial \nu}\right], \tag{6.8}$$

$$\sigma_n = C_1 (c_1 - c_2) + C_2 [(c_1 + c_2) c_3 - 2 c_2] x_n^{c_3 - 1} - \frac{2 C_3 (c_1 + 2 c_2)}{x_n^3}, \tag{6.9}$$

$$\sigma_\varphi = \sigma_\theta = C_1 (c_1 - c_2) + C_2 (c_1 - c_2 c_3) x_n^{c_3 - 1} + \frac{C_3 (c_1 + 2 c_2)}{x_n^3}, \tag{6.10}$$

$$w = \kappa_1 + \kappa_2 x_n^{2(c_3 - 1)} + \frac{\kappa_3}{x_n^6} + \kappa_4 x_n^{c_3 - 1} + \frac{\kappa_5}{x_n^3} + \kappa_6 x_n^{c_3 - 4}, \tag{6.11}$$

where Θ is given by Equation (1.21), and κ_i $(i = 1, \ldots, 6)$ is derived as

$$\begin{aligned}
\kappa_1 &= \frac{3 (c_1 - c_2) C_1^2}{2} + \frac{1}{s_{44}} \left[\left(\frac{\partial C_1}{\partial \varphi} \right)^2 + \Theta^2 \left(\frac{\partial C_1}{\partial \nu} \right)^2 \right], \\
\kappa_2 &= \left[\frac{(c_1 + c_2) c_3^2}{2} + c_1 - 2 c_2 c_3 \right] C_2^2 + \frac{1}{s_{44}} \left[\left(\frac{\partial C_2}{\partial \varphi} \right)^2 + \Theta^2 \left(\frac{\partial C_2}{\partial \nu} \right)^2 \right], \\
\kappa_3 &= 3 (c_1 + 2 c_2) C_3^2 + \frac{1}{s_{44}} \left[\left(\frac{\partial C_3}{\partial \varphi} \right)^2 + \Theta^2 \left(\frac{\partial C_3}{\partial \nu} \right)^2 \right], \\
\kappa_4 &= (c_1 - c_2)(2 + c_3) C_1 C_2 + \frac{2}{s_{44}} \left(\frac{\partial C_1}{\partial \varphi} \frac{\partial C_2}{\partial \varphi} + \Theta^2 \frac{\partial C_1}{\partial \nu} \frac{\partial C_2}{\partial \nu} \right), \\
\kappa_5 &= \frac{2}{s_{44}} \left(\frac{\partial C_1}{\partial \varphi} \frac{\partial C_3}{\partial \varphi} + \Theta^2 \frac{\partial C_1}{\partial \nu} \frac{\partial C_3}{\partial \nu} \right), \\
\kappa_6 &= [2 c_2 (1 - c_3) - c_1] C_2 C_3 + \frac{2}{s_{44}} \left(\frac{\partial C_2}{\partial \varphi} \frac{\partial C_3}{\partial \varphi} + \Theta^2 \frac{\partial C_2}{\partial \nu} \frac{\partial C_3}{\partial \nu} \right).
\end{aligned} \tag{6.12}$$

6.2 Condition $\beta_{IN} \neq \beta_E \neq \beta_M$

6.2.1 Matrix

The integration constants C_{1M}, C_{2M}, C_{3M} for the matrix (see Equation (6.4)) are determined by the boundary conditions (2.29), (2.30) or (2.29)–(2.32). The boundary conditions result in the following combinations of C_{1M}, C_{2M}, C_{3M}. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$. With regard to Equations (2.29), (2.30), (6.4)–(6.11), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M}\left[1 - c_{3M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{6.13}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[1 - \left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{6.14}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right) - x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right], \tag{6.15}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right) - x_n^{c_{3M}-1}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right], \tag{6.16}$$

$$\sigma_{nM} = -\frac{p_{2n}}{\zeta_M}\left\{c_{1M} - c_{2M} - \left[c_{3M}\left(c_{1M}+c_{2M}\right) - 2c_{2M}\right]\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right\}, \tag{6.17}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[c_{1M} - c_{2M} - \left(c_{1M} - c_{2M}c_{3M}\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{6.18}$$

$$w_M = \frac{1}{2}\left(\kappa_{1M} + \kappa_{2M}\,x_n^{2(c_{3M}-1)} + \kappa_{4M}\,x_n^{c_{3M}-1}\right), \tag{6.19}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \frac{\kappa_{2M}}{2c_{3M}+1}\left(x_M^{2c_{3M}+1} - x_E^{2c_{3M}+1}\right)\right.$$
$$\left. + \frac{\kappa_{4M}}{c_{3M}+2}\left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2}\right)\right] d\varphi\, d\nu, \tag{6.20}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i=1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_M, κ_{iM} $(i=1,2,4$; see Equation (6.12)) have the forms

$$\zeta_M = c_{1M} - c_{2M} - \left[c_{3M}\left(c_{1M}+c_{2M}\right) - 2c_{2M}\right]\left(\frac{x_E}{x_M}\right)^{c_{3M}-1},$$

$$\kappa_{1M} = \frac{3\left(c_{1M}-c_{2M}\right)}{2}\left(\frac{p_{2n}}{\zeta_M}\right)^2 + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2\right\},$$

$$\kappa_{2M} = \left[\frac{\left(c_{1M}+c_{2M}\right)c_{3M}^2}{2} + c_{1M} - 2c_{2M}c_{3M}\right]\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)^2$$

$$+\frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2\right\},$$

$$\kappa_{4M}=\frac{(c_{2M}-c_{1M})(2+c_{3M})}{x_M^{c_{3M}-1}}\left(\frac{p_{2n}}{\zeta_M}\right)^2$$

$$-\frac{2}{s_{44M}}\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)+\Theta^2\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]. \tag{6.21}$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i,j=1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E}=-p_{2n}\,\rho_M$ and Equation (6.14), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M=\frac{1}{\zeta_M}\left[1-\left(\frac{x_E}{x_M}\right)^{c_{3M}-1}\right]. \tag{6.22}$$

Conditions $C_{1M}\neq 0$, $C_{3M}\neq 0$, $C_{2M}=0$. With regard to Equations (2.29), (2.30), (6.5)–(6.11), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM}=-\frac{p_{2n}}{\zeta_M}\left[1-c_{3M}\left(\frac{x_M}{x_n}\right)^3\right], \tag{6.23}$$

$$\varepsilon_{\varphi M}=\varepsilon_{\theta M}=-\frac{p_{2n}}{\zeta_M}\left[1-\left(\frac{x_M}{x_n}\right)^3\right], \tag{6.24}$$

$$\varepsilon_{n\varphi M}=s_{44M}\,\sigma_{n\varphi M}=-\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)-\frac{1}{x_n^3}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)\right], \tag{6.25}$$

$$\varepsilon_{n\theta M}=s_{44M}\,\sigma_{n\theta M}=-\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)-\frac{1}{x_n^3}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)\right], \tag{6.26}$$

$$\sigma_{nM}=-\frac{p_{2n}}{\zeta_M}\left[c_{1M}-c_{2M}+2\left(c_{1M}+2c_{2M}\right)\left(\frac{x_M}{x_n}\right)^3\right], \tag{6.27}$$

$$\sigma_{\varphi M}=\sigma_{\theta M}=-\frac{p_{2n}}{\zeta_M}\left[c_{1M}-c_{2M}-\left(c_{1M}+2c_{2M}\right)\left(\frac{x_M}{x_n}\right)^3\right], \tag{6.28}$$

$$w_M = \kappa_{1M} + \frac{\kappa_{3M}}{x_n^6} + \frac{\kappa_{5M}}{x_n^3}, \tag{6.29}$$

$$W_M = 4 \int_0^{\pi/2} \int_0^{\pi/2} \left[\frac{\kappa_{1M}}{3} \left(x_M^3 - x_E^3 \right) + \frac{\kappa_{3M}}{3} \left(\frac{1}{x_E^3} - \frac{1}{x_M^3} \right) + \kappa_{5M} \ln \left(\frac{x_M}{x_E} \right) \right] d\varphi d\nu, \tag{6.30}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i=1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_M, κ_{iM} $(i=3,5$; see Equation (6.12)) have the forms

$$\zeta_M = c_{1M} - c_{2M} + 2\left(c_{1M} + 2c_{2M}\right) \left(\frac{x_M}{x_E} \right)^3,$$
$$\kappa_{3M} = 3\left(c_{1M} + 2c_{2M}\right) \left(\frac{p_{2n} x_M^3}{\zeta_M} \right)^2 + \frac{1}{s_{44M}} \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n} x_M^3}{\zeta_M} \right) \right]^2 + \frac{\Theta^2}{s_{44M}} \left[\frac{\partial}{\partial \nu} \left(\frac{p_{2n} x_M^3}{\zeta_M} \right) \right]^2,$$
$$\kappa_{5M} = -\frac{2}{s_{44M}} \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}}{\zeta_M} \right) \frac{\partial}{\partial \varphi} \left(\frac{p_{2n} x_M^3}{\zeta_M} \right) + \Theta^2 \frac{\partial}{\partial \nu} \left(\frac{p_{2n}}{\zeta_M} \right) \frac{\partial}{\partial \nu} \left(\frac{p_{2n} x_M^3}{\zeta_M} \right) \right]. \tag{6.31}$$

The coefficient κ_{1M} is given by Equation (6.21), where ζ_M in Equation (6.21) is given by Equation (6.31). The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i,j=1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n = x_E} = -p_{2n}\rho_M$ and Equation (6.24), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M = \frac{1}{\zeta_M} \left[1 - \left(\frac{x_M}{x_E} \right)^{-3} \right]. \tag{6.32}$$

Conditions $C_{2M} \neq 0$, $C_{3M} \neq 0$, $C_{1M} = 0$. With regard to Equations (2.29), (2.30), (6.5)–(6.11), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M} \left[c_{3M} \left(\frac{x_n}{x_M} \right)^{c_{3M}-1} - 2 \left(\frac{x_M}{x_n} \right)^3 \right], \tag{6.33}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M} \left[\left(\frac{x_n}{x_M} \right)^{c_{3M}-1} - \left(\frac{x_M}{x_n} \right)^3 \right], \tag{6.34}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right) x_n^{c_{3M}-1} - \frac{1}{x_n^3}\,\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right)\right], \quad (6.35)$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right) x_n^{c_{3M}-1} - \frac{1}{x_n^3}\,\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right)\right], \quad (6.36)$$

$$\sigma_{nM} = -\frac{p_{2n}}{\zeta_M}\left\{\left[c_{3M}\left(c_{1M}+c_{2M}\right)-2c_{2M}\right]\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} - 2\left(c_{1M}+2c_{2M}\right)\left(\frac{x_M}{x_n}\right)^3\right\}, \quad (6.37)$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\left(c_{1M}-c_{2M}c_{3M}\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} + \left(c_{1M}+2c_{2M}\right)\left(\frac{x_M}{x_n}\right)^3\right], \quad (6.38)$$

$$w_M = \kappa_{2M}\, x_n^{2(c_{3M}-1)} + \frac{\kappa_{3M}}{x_n^6} + \kappa_{6M}\, x_n^{c_{3M}-4}, \quad (6.39)$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{2M}}{2c_{3M}+1}\left(x_M^{2c_{3M}+1} - x_E^{2c_{3M}+1}\right) + \frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right)\right.$$
$$\left. + \frac{\kappa_{6M}}{c_{3M}-1}\left(x_M^{c_{3M}-1} - x_E^{c_{3M}-1}\right)\right] d\varphi\, d\nu, \quad (6.40)$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i = 1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_M, κ_{6M} (see Equation (6.12)) have the forms

$$\zeta_M = \left\{\left[c_{3M}\left(c_{1M}+c_{2M}\right)-2c_{2M}\right]\left(\frac{x_E}{x_M}\right)^{c_{3M}+2} + 2\left(c_{1M}+2c_{2M}\right)\right\}\left(\frac{x_M}{x_E}\right)^3,$$

$$\kappa_{6M} = -\frac{x_M^3\left[2c_{2M}\left(1-c_{3M}\right)-c_{1M}\right]}{x_M^{c_{3M}-1}}\left(\frac{p_{2n}}{\zeta_M}\right)^2$$
$$- \frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right)$$
$$- \frac{2\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right). \quad (6.41)$$

The coefficients κ_{2M}, κ_{3M} are given by Equations (6.21), (6.31), respectively, where ζ_M in Equations (6.21), (6.31) is given by Equation (6.41). The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i, j = 1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n = x_E} = -p_{2n}\,\rho_M$ and Equation (6.34), the coefficient ρ_M in Equation (2.26) is derived as

$$\rho_M = \frac{1}{\zeta_M}\left[\left(\frac{x_E}{x_M}\right)^{c_{3M}-1} - \left(\frac{x_M}{x_E}\right)^3\right]. \tag{6.42}$$

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} \neq 0$. With regard to Equations (2.29)–(2.32), (6.5)–(6.11), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M}\left\{1 - \frac{1}{c_{3M}+2}\left[3c_{3M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} - 2\left(c_{3M}-1\right)\left(\frac{x_M}{x_n}\right)^3\right]\right\}, \tag{6.43}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left\{1 - \frac{1}{c_{3M}+2}\left[3\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} + \left(c_{3M}-1\right)\left(\frac{x_M}{x_n}\right)^3\right]\right\}, \tag{6.44}$$

$$\begin{aligned}\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = &-\left\{\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right.\\ &\left.- \frac{1}{c_{3M}+2}\left[3\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)x_n^{c_{3M}-1} + \frac{c_{3M}-1}{x_n^3}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)\right]\right\},\end{aligned} \tag{6.45}$$

$$\begin{aligned}\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = &-\Theta\left\{\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right.\\ &\left.- \frac{1}{c_{3M}+2}\left[3\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)x_n^{c_{3M}-1} + \frac{c_{3M}-1}{x_n^3}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)\right]\right\},\end{aligned} \tag{6.46}$$

$$\begin{aligned}\sigma_{nM} = -\frac{p_{2n}}{\zeta_M}&\left\{c_{1M} - c_{2M} - \frac{3}{c_{3M}+2}\left[c_{3M}\left(c_{1M}+c_{2M}\right) - 2c_{2M}\right]\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right.\\ &\left.+\frac{2\left(c_{1M}+2c_{2M}\right)}{c_{3M}+2}\left(\frac{x_M}{x_n}\right)^3\right\},\end{aligned} \tag{6.47}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left\{c_{1M} - c_{2M} - \frac{3\left(c_{1M} - c_{2M}c_{3M}\right)}{c_{3M}+2}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} - \frac{c_{1M}+2c_{2M}}{c_{3M}+2}\left(\frac{x_M}{x_n}\right)^3\right\}, \tag{6.48}$$

$$w_M = \kappa_{1M} + \kappa_{2M}\, x_n^{2(c_{3M}-1)} + \frac{\kappa_{3M}}{x_n^6} + \kappa_{4M}\, x_n^{c_{3M}-1} + \frac{\kappa_{5M}}{x_n^3} + \kappa_{6M}\, x_n^{c_{3M}-4}, \tag{6.49}$$

$$\begin{aligned} W_M = 4\int_0^{\pi/2}\int_0^{\pi/2} \Bigg[& \frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \frac{\kappa_{2M}}{2c_{3M}+1}\left(x_M^{2c_{3M}+1} - x_E^{2c_{3M}+1}\right) \\ & + \frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) + \frac{\kappa_{4M}}{c_{3M}+2}\left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2}\right) \\ & + \kappa_{5M}\ln\left(\frac{x_M}{x_E}\right) + \frac{\kappa_{6M}}{c_{3M}-1}\left(x_M^{c_{3M}-1} - x_E^{c_{3M}-1}\right)\Bigg]\, d\varphi\, d\nu, \end{aligned} \tag{6.50}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} ($i = 1,2,3$) are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_M, κ_{iM} ($i = 2\ldots, 6$; see Equation (6.12)) have the forms

$$\begin{aligned} \zeta_M = c_{1M} - c_{2M} + \frac{1}{c_{3M}+2}\left(\frac{x_M}{x_E}\right)^3 \Bigg\{ & 2\left(c_{3M}-1\right)\left(c_{1M}+2c_{Mm}\right) \\ & -3\left[c_{3M}\left(c_{1M}+c_{2M}\right) - 2c_{2M}\right]\left(\frac{x_E}{x_M}\right)^{c_{3M}+2}\Bigg\}, \end{aligned}$$

$$\begin{aligned} \kappa_{2M} = \left(\frac{3}{c_{3M}+2}\right)^2 \Bigg(& \left[\frac{\left(c_{1M}+c_{2M}\right)c_{3M}^2}{2} + c_{1M} - 2c_{2M}c_{3M}\right]\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)^2 \\ & + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2\right\}\Bigg), \end{aligned}$$

$$\begin{aligned} \kappa_{3M} = \left(\frac{c_{3M}-1}{c_{3M}+2}\right)^2 \Bigg(& 3\left(c_{1M}+2c_{2M}\right)\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)^2 \\ & + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)\right]^2\right\}\Bigg), \end{aligned}$$

$$
\begin{aligned}
\kappa_{4M} &= \frac{3\left(c_{2M}-c_{1M}\right)}{x_M^{c_{3M}-1}}\left(\frac{p_{2n}}{\zeta_M}\right)^2 \\
&- \frac{6}{s_{44M}\left(c_{3M}+2\right)} \\
&\quad \times \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)+\Theta^2\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right], \\
\kappa_{5M} &= \frac{2\left(1-c_{3M}\right)}{s_{44M}\left(c_{3M}+2\right)}\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)+\Theta^2\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)\right], \\
\kappa_{6M} &= \frac{3\left(c_{3M}-1\right)\left[2c_{2M}\left(1-c_{3M}\right)-c_{1M}\right]}{x_M^{c_{3M}-4}}\left[\frac{p_{2n}}{\zeta_M\left(c_{3M}+2\right)}\right]^2 \\
&+ \frac{6\left(c_{3M}-1\right)}{s_{44M}\left(c_{3M}+2\right)^2}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right) \\
&+ \frac{6\Theta^2\left(c_{3M}-1\right)}{s_{44M}\left(c_{3M}+2\right)^2}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right).
\end{aligned}
\tag{6.51}
$$

The coefficient κ_{1M} is given by Equation (6.21), where ζ_M in Equation (6.21) is given by Equation (6.51). The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i, j = 1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi M}\right)_{x_n=x_E} = -p_{2n}\rho_M$ and Equation (6.44), the coefficient ρ_M in Equation (2.26) is derived as

$$
\rho_M = \frac{1}{\zeta_M}\left\{1-\frac{1}{c_{3M}+2}\left[3\left(\frac{x_E}{x_M}\right)^{c_{3M}-1}+\left(c_{3M}-1\right)\left(\frac{x_M}{x_E}\right)^3\right]\right\}. \tag{6.52}
$$

6.2.2 Envelope

The integration constants C_{1E}, C_{2E}, C_{3E} for the envelope (see Equation (6.4)) are determined by the boundary conditions (2.33), (2.34). The boundary conditions result in the following combinations of C_{1E}, C_{2E}, C_{3E}, i.e., $C_{1E} \neq 0$, $C_{2E} \neq 0$, $C_{3E} = 0$; $C_{1E} \neq 0$, $C_{3E} \neq 0$, $C_{2E} = 0$; $C_{2E} \neq 0$, $C_{3E} \neq 0$, $C_{1E} = 0$. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1E} \neq 0$, $C_{2E} \neq 0$, $C_{3E} = 0$. With regard to Equations (2.33), (2.34), (6.4)–(6.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\left[\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}} + \frac{c_{3E}\,(p_{1n} - p_{2n})}{\zeta_{3E}}\, x_n^{c_{3E}-1}\right], \tag{6.53}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\left[\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}} + \frac{p_{1n} - p_{2n}}{\zeta_{3E}}\, x_n^{c_{3E}-1}\right], \tag{6.54}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + x_n^{c_{3E}-1}\,\frac{\partial}{\partial\varphi}\left(\frac{p_{1n} - p_{2n}}{\zeta_{3E}}\right)\right], \tag{6.55}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\,\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + x_n^{c_{3E}-1}\,\frac{\partial}{\partial\nu}\left(\frac{p_{1n} - p_{2n}}{\zeta_{3E}}\right)\right], \tag{6.56}$$

$$\begin{aligned}\sigma_{nE} = -\Bigg\{&\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)(c_{1E} - c_{2E}) \\ &+ \frac{(p_{1n} - p_{2n})\left[(c_{1E} + c_{2E})\,c_{3E} - 2\,c_{2E}\right]}{\zeta_{3E}}\, x_n^{c_{3E}-1}\Bigg\},\end{aligned} \tag{6.57}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = (c_{1E} - c_{2E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{(c_{1E} - c_{2E}\,c_{3E})\,(p_{1n} - p_{2n})}{\zeta_{3E}} x_n^{c_{3E}-1}, \tag{6.58}$$

$$w_E = \kappa_{1E} + \kappa_{2E}\, x_n^{2(c_{3E}-1)} + \kappa_{4E}\, x_n^{c_{3E}-1}, \tag{6.59}$$

$$\begin{aligned}W_E = 4\int_0^{\pi/2}\int_0^{\pi/2} &\left[\frac{\kappa_{1E}}{3}\left(x_E^3 - x_{IN}^3\right) + \frac{\kappa_{2E}}{2c_{3E}+1}\left(x_E^{2c_{3E}+1} - x_{IN}^{2c_{3C}+1}\right)\right. \\ &\left. + \frac{\kappa_{3E}}{c_{3e}+2}\left(x_e^{c_{3e}+2} - x_{IN}^{c_{3E}+2}\right)\right] d\varphi\, d\nu,\end{aligned} \tag{6.60}$$

where Θ, x_{IN}, x_E and s_{44E}, c_{iE} $(i = 1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_{iE} $(i = 1,2,3)$, κ_{jE} $(j = 1,2,4$; see Equation (6.12)) have the forms

$$\zeta_{1E} = (c_{1E} - c_{2E})\left[1 - \left(\frac{x_{IN}}{x_E}\right)^{c_{3E}-1}\right],$$

$$\zeta_{2E} = (c_{1E} - c_{2E})\left[\left(\frac{x_E}{x_{IN}}\right)^{c_{3E}-1} - 1\right],$$

$$\zeta_{3E} = [c_{3E}(c_{1E}+c_{2E}) - 2c_{2E}]\left(x_{IN}^{c_{3E}-1} - x_E^{c_{3E}-1}\right),$$

$$\kappa_{1E} = \frac{3(c_{1E}-c_{2E})}{2}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)^2 + \frac{1}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\right]^2\right\},$$

$$\kappa_{2E} = \left[\frac{(c_{1E}+c_{2E})c_{3E}^2}{2} + c_{1E} - 2c_{2E}c_{3E}\right]\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right)^2 + \frac{1}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right)\right]^2\right\},$$

$$\kappa_{4E} = (c_{1E}-c_{2E})(2+c_{3E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right) + \frac{2}{s_{44E}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right) + \frac{2\Theta^2}{s_{44E}}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right), \tag{6.61}$$

The normal stresses p_{1n}, p_{2n} are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (6.22), (6.32), (6.42), (6.52), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}} = -(p_{1n}\rho_{11E} + p_{2n}\rho_{12E})$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E} = -(p_{1n}\rho_{21E} + p_{2n}\rho_{22E})$ and Equation (6.54), the coefficient ρ_{ijE} ($i, j = 1,2$) in Equations (2.25), (2.26) is derived as

$$\rho_{1iE} = (\delta_{1i} - \delta_{2i})\left(\frac{1}{\zeta_{iE}} + \frac{x_{IN}^{c_{3E}-1}}{\zeta_{3E}}\right), \quad \rho_{2iE} = (\delta_{1i} - \delta_{2i})\left(\frac{1}{\zeta_{iE}} + \frac{x_E^{c_{3E}-1}}{\zeta_{3E}}\right), \quad i = 1,2, \tag{6.62}$$

where δ_{ij} represents Kronecker's delta [24], i.e., $\delta_{ij} = 0$ or $\delta_{ij} = 1$ for $i \neq j$ or $i = j$, respectively.

Conditions $C_{1E} \neq 0$, $C_{3E} \neq 0$, $C_{2E} = 0$. With regard to Equations (2.33), (2.34), (6.4)–(6.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}} - \frac{2(p_{1n}-p_{2n})}{\zeta_{3E}x_n^3}\right), \tag{6.63}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}} + \frac{p_{1n}-p_{2n}}{\zeta_{3E}x_n^3}\right), \tag{6.64}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{1}{x_n^3}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right)\right], \tag{6.65}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\,\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{1}{x_n^3}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right)\right], \tag{6.66}$$

$$\sigma_{nE} = -\left[(c_{1E}-c_{2E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) - \frac{2\,(c_{1E}+2\,c_{2E})\,(p_{1n}-p_{2n})}{\zeta_{3E}\,x_n^3}\right], \tag{6.67}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = \left[(c_{1E}-c_{2E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{(c_{1E}+2\,c_{2E})\,(p_{1n}-p_{2n})}{\zeta_{3E}\,x_n^3}\right], \tag{6.68}$$

$$w_E = \kappa_{1E} + \frac{\kappa_{3E}}{x_n^6} + \frac{\kappa_{5E}}{x_n^3}, \tag{6.69}$$

$$W_E = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{1E}}{3}\left(x_E^3 - x_{IN}^3\right) + \frac{\kappa_{3E}}{3}\left(\frac{1}{x_{IN}^3} - \frac{1}{x_E^3}\right) + \kappa_{5E}\ln\left(\frac{x_E}{x_{IN}}\right)\right] d\varphi\, d\nu, \tag{6.70}$$

where Θ, x_{IN}, x_E and s_{44E}, c_{iE} $(i=1{,}2{,}3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_{iE}, κ_{iE} ($i=3{,}5$; see Equation (6.12)) have the forms

$$\begin{aligned}
\zeta_{1E} &= (c_{1E}-c_{2E})\left[1-\left(\frac{x_E}{x_{IN}}\right)^3\right],\\
\zeta_{2E} &= (c_{1E}-c_{2E})\left[\left(\frac{x_{IN}}{x_E}\right)^3 - 1\right],\\
\zeta_{3E} &= 2\,(c_{1E}+2c_{2E})\left(\frac{1}{x_{IN}^3} - \frac{1}{x_E^3}\right),\\
\kappa_{3E} &= 3\,(c_{1E}+2\,c_{2E})\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right)^2\\
&\quad + \frac{1}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right)\right]^2\right\},\\
\kappa_{5E} &= \frac{2}{s_{44E}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right)\\
&\quad + \frac{2\,\Theta^2}{s_{44E}}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{1n}-p_{2n}}{\zeta_{3E}}\right).
\end{aligned} \tag{6.71}$$

The coefficient κ_{1E} is given by Equation (6.61), where ζ_M in Equation (6.61) is given by Equation (6.71). The normal stresses p_{1n}, p_{2n} are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (6.22), (6.32), (6.42), (6.52), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n = x_{IN}} = -\left(p_{1n}\,\rho_{11E} + p_{2n}\,\rho_{12E}\right)$, $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E} = -\left(p_{1n}\,\rho_{21E} + p_{2n}\,\rho_{22E}\right)$ and Equation (6.64), the coefficient ρ_{ijE} $(i, j = 1{,}2)$ in Equations (2.25), (2.26) is derived as

$$\rho_{1iE} = \left(\delta_{1i} - \delta_{2i}\right)\left(\frac{1}{\zeta_{iE}} + \frac{1}{\zeta_{3E}\,x_{IN}^3}\right), \quad \rho_{2iE} = \left(\delta_{1i} - \delta_{2i}\right)\left(\frac{1}{\zeta_{iE}} + \frac{1}{\zeta_{3E}\,x_E^3}\right),$$
$$i = 1, 2, \tag{6.72}$$

where δ_{ij} represents Kronecker's delta [24], i.e., $\delta_{ij} = 0$ or $\delta_{ij} = 1$ for $i \neq j$ or $i = j$, respectively.

Conditions $C_{1E} \neq 0$**,** $C_{3E} \neq 0$**,** $C_{2E} = 0$**.** With regard to Equations (2.33), (2.34), (6.4)–(6.11), (2.22), (2.23), we get

$$\varepsilon_n = C_2\,c_{3E}\,x_n^{c_{3E}-1} - \frac{2\,C_3}{x_n^3}, \tag{6.73}$$

$$\varepsilon_\varphi = \varepsilon_\theta = C_2\,x_n^{c_{3E}-1} + \frac{C_3}{x_n^3}, \tag{6.74}$$

$$\varepsilon_{n\varphi} = s_{44E}\,\sigma_{n\varphi} = x_n^{c_{3E}-1}\,\frac{\partial C_2}{\partial \varphi} + \frac{1}{x_n^3}\,\frac{\partial C_3}{\partial \varphi}, \tag{6.75}$$

$$\varepsilon_{n\theta} = s_{44E}\,\sigma_{n\theta} = \Theta\left[x_n^{c_{3E}-1}\,\frac{\partial C_2}{\partial \nu} + \frac{1}{x_n^3}\,\frac{\partial C_3}{\partial \nu}\right], \tag{6.76}$$

$$\sigma_n = C_2\left[\left(c_{1E} + c_{2E}\right)c_{3E} - 2\,c_{2E}\right]x_n^{c_{3E}-1} - \frac{2\,C_3\left(c_{1E} + 2\,c_{2E}\right)}{x_n^3}, \tag{6.77}$$

$$\sigma_\varphi = \sigma_\theta = C_2\left(c_{1E} - c_{2E}\,c_{3E}\right)x_n^{c_{3E}-1} + \frac{C_3\left(c_{1E} + 2\,c_{2E}\right)}{x_n^3}, \tag{6.78}$$

$$w_E = \kappa_{2E}\,x_n^{2\left(c_{3E}-1\right)} + \frac{\kappa_{3E}}{x_n^6} + \kappa_{6E}\,x_n^{c_{3E}-4}, \tag{6.79}$$

$$W_E = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{2E}}{2c_{3E}+1}\left(x_E^{2c_{3E}+1} - x_{IN}^{2c_{3C}+1}\right) + \frac{\kappa_{3E}}{3}\left(\frac{1}{x_{IN}^3} - \frac{1}{x_E^3}\right)\right.$$
$$\left. + \frac{\kappa_{6E}}{c_{3E}-1}\left(x_E^{c_{3E}-1} - x_{IN}^{c_{3C}-1}\right)\right]d\varphi\,d\nu, \tag{6.80}$$

where Θ, x_{IN}, x_E and s_{44E}, c_{iE} $(i=1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_{iE} $(i=1,\ldots,4)$, κ_{jM} ($j=2,3,6$; see Equation (6.12)) have the forms

$$\begin{aligned}
\zeta_{1E} &= x_{IN}^{c_{3E}-1}\left[c_{3E}\left(c_{1E}+c_{2E}\right)-2c_{2E}\right]\left[1-\left(\frac{x_E}{x_{IN}}\right)^{c_{3E}+2}\right],\\
\zeta_{2E} &= x_{E}^{c_{3E}-1}\left[c_{3E}\left(c_{1E}+c_{2E}\right)-2c_{2E}\right]\left[\left(\frac{x_{IN}}{x_E}\right)^{c_{3E}+2}-1\right],\\
\zeta_{3E} &= \frac{2\left(c_{1E}+2c_{2E}\right)}{x_{IN}^3}\left[\left(\frac{x_{IN}}{x_E}\right)^{c_{3E}+2}-1\right],\\
\zeta_{3E} &= \frac{2\left(c_{1E}+2c_{2E}\right)}{x_{E}^3}\left[1-\left(\frac{x_{E}}{x_{IN}}\right)^{c_{3E}+2}\right],\\
\kappa_{2E} &= \left[\frac{\left(c_{1E}+c_{2E}\right)c_{3E}^2}{2}+c_{1E}-2\,c_{2E}\,c_{3E}\right]\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)^2\\
&\quad+\frac{1}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\right]^2\right\},\\
\kappa_{3E} &= 3\left(c_{1E}+2\,c_{2E}\right)\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)^2\\
&\quad+\frac{1}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right)\right]^2\right\},\\
\kappa_{6E} &= \left[2\,c_{2E}\left(1-c_{3E}\right)-c_{1E}\right]\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right)\\
&\quad+\frac{2}{s_{44E}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right)\\
&\quad+\frac{2\Theta^2}{s_{44E}}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right).
\end{aligned} \tag{6.81}$$

The normal stresses p_{1n}, p_{2n} are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (6.22), (6.32), (6.42), (6.52), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}}=-\left(p_{1n}\,\rho_{11E}+p_{2n}\,\rho_{12E}\right)$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E}=-\left(p_{1n}\,\rho_{21E}+p_{2n}\,\rho_{22E}\right)$ and Equation (6.74), the coefficient ρ_{ijE} $(i,j=1,2)$ in Equations (2.25), (2.26) is derived as

$$\rho_{1iE} = (\delta_{1i} - \delta_{2i}) \left(\frac{x_{IN}^{c_{3E}-1}}{\zeta_{iE}} + \frac{1}{\zeta_{3E}\, x_{IN}^3} \right), \quad \rho_{2iE} = (\delta_{1i} - \delta_{2i}) \left(\frac{x_{E}^{c_{3E}-1}}{\zeta_{iE}} + \frac{1}{\zeta_{3E}\, x_{E}^3} \right),$$
$$i = 1, 2, \tag{6.82}$$

where δ_{ij} represents Kronecker's delta [24], i.e., $\delta_{ij} = 0$ or $\delta_{ij} = 1$ for $i \neq j$ or $i = j$, respectively.

6.2.3 Inclusion

In case of the ellipsoidal inclusion, we get $C_{2IN} = C_{3IN} = 0$, otherwise we get $(u_{nIN})_{x_n \to 0} \longrightarrow \pm\infty$, $(\varepsilon_{IN})_{x_n \longrightarrow 0} \longrightarrow \pm\infty$, $(\sigma_{IN})_{r \to 0} \longrightarrow \pm\infty$ due to $c_3 < 0$ (see Equations (2.18), (6.4)–(6.10)). With regard to Equations (2.35), (2.36), (6.4)–(6.11), (2.21), (2.22), (2.23), we get [2]–[23]

$$\varepsilon_{nIN} = \varepsilon_{\varphi IN} = \varepsilon_{\theta IN} = -p_{1n}\, \rho_{IN}, \tag{6.83}$$

$$\varepsilon_{n\varphi IN} = s_{44IN}\, \sigma_{n\varphi IN} = -\rho_{IN}\, \frac{\partial p_{1n}}{\partial \varphi}, \tag{6.84}$$

$$\varepsilon_{n\theta IN} = s_{44IN}\, \sigma_{n\theta IN} = -\Theta\, \rho_{IN}\, \frac{\partial p_{1n}}{\partial \nu}, \tag{6.85}$$

$$\sigma_{nIN} = \sigma_{\varphi IN} = \sigma_{\theta IN} = -p_{1n}, \tag{6.86}$$

$$w_{IN} = \frac{\rho_{IN}^2}{2} \left\{ \frac{3 p_{1n}^2}{2 \rho_{IN}} + \frac{2}{s_{44IN}} \left[\left(\frac{\partial p_{1n}}{\partial \varphi} \right)^2 + \left(\frac{\partial p_{1n}}{\partial \nu} \right)^2 \right] \right\}, \tag{6.87}$$

$$W_{IN} = \frac{4 \rho_{IN}^2}{3} \int_0^{\pi/2} \int_0^{\pi/2} x_{IN}^3 \left\{ \frac{3 p_{1n}^2}{2 \rho_{IN}} + \frac{2}{s_{44IN}} \left[\left(\frac{\partial p_{1n}}{\partial \varphi} \right)^2 + \left(\frac{\partial p_{1n}}{\partial \nu} \right)^2 \right] \right\} d\varphi\, d\nu, \tag{6.88}$$

where Θ, s_{44IN} are given by Equations (1.21), (2.13), respectively. The normal stress p_{1n} is given by Equation (2.25). With regard to $\left(\varepsilon_{\varphi IN}\right)_{x_n = x_{IN}} = -p_{1n}\, \rho_{IN}$ and Equation (6.83), the coefficient ρ_{IN} in Equation (2.26) is derived as

$$\rho_{IN} = \frac{1 - 2\mu_{IN}}{E_{IN}}. \tag{6.89}$$

6.3 Condition $\beta_{IN} \neq \beta_E = \beta_M$

If $\beta_{IN} \neq \beta_E = \beta_M$, then the thermal strains and stresses in the ellipsoidal inclusion are determined in Section 6.2.3. The normal stress p_{1n} in Section 6.2.3 is given by Equation (2.27), where ρ_{IN} and ρ_{1ME} in Equation (2.27) are given by Equations (6.89) and (6.135), respectively.

6.3.1 Matrix

The integration constants C_{1M}, C_{2M}, C_{3M} for the matrix (see Equation (5.4)) are determined by the boundary conditions (2.30), (2.31) or (2.30)–(2.32). The boundary conditions result in the following combinations of C_{1M}, C_{2M}, C_{3M}, i.e., $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$; $C_{1M} \neq 0$, $C_{3M} \neq 0$, $C_{2M} = 0$; $C_{2M} \neq 0$, $C_{3M} \neq 0$, $C_{1M} = 0$; $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} \neq 0$. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} = 0$. With regard to Equations (2.30), (2.31), (6.4)–(6.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}\left[1 - c_{3M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{6.90}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}\left[1 - \left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{6.91}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right) - x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right], \tag{6.92}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right) - x_n^{c_{3M}-1}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right], \tag{6.93}$$

$$\sigma_{nM} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}\left\{c_{1M} - c_{2M} - \left[c_{3M}\left(c_{1M} + c_{2M}\right) - 2c_{2M}\right]\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right\}, \tag{6.94}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}\left[c_{1M} - c_{2M} - \left(c_{1M} - c_{2M}c_{3M}\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{6.95}$$

$$w_M = \frac{1}{2}\left(\kappa_{1M} + \kappa_{2M} x_n^{2(c_{3M}-1)} + \kappa_{4M} x_n^{c_{3M}-1}\right), \tag{6.96}$$

$$\begin{aligned} W_M = 4 \int_0^{\pi/2}\int_0^{\pi/2} \Bigg[& \frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \frac{\kappa_{2M}}{2c_{3M}+1}\left(x_M^{2c_{3M}+1} - x_E^{2c_{3M}+1}\right) \\ & + \frac{\kappa_{4M}}{c_{3M}+2}\left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2}\right)\Bigg] d\varphi\, d\nu, \end{aligned} \tag{6.97}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i=1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_M, κ_{iM} $(i=1,2,4$; see Equation (6.12)) have the forms

$$\begin{aligned} \zeta_M &= 1 - \left(\frac{x_E}{x_M}\right)^{c_{3M}-1}, \\ \kappa_{1M} &= \frac{3\left(c_{1M}-c_{2M}\right)}{2}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)^2 \\ &\quad + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)\right]^2\right\}, \\ \kappa_{2M} &= \left[\frac{\left(c_{1M}+c_{2M}\right)c_{3M}^2}{2} + c_{1M} - 2c_{2M}c_{3M}\right]\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M x_M^{c_{3M}-1}}\right)^2 \\ &\quad + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M x_M^{c_{3M}-1}}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M x_M^{c_{3M}-1}}\right)\right]^2\right\}, \\ \kappa_{4M} &= \frac{\left(c_{2M}-c_{1M}\right)\left(2+c_{3M}\right)}{x_M^{c_{3M}-1}}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)^2 \\ &\quad - \frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M x_M^{c_{3M}-1}}\right) \\ &\quad - \frac{2\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M x_M^{c_{3M}-1}}\right). \end{aligned} \tag{6.98}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{3M} \neq 0$, $C_{2M} = 0$. With regard to Equations (2.30), (2.31), (6.4)–(6.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\left[1 - c_{3M}\left(\frac{x_M}{x_n}\right)^3\right], \tag{6.99}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\left[1 - \left(\frac{x_M}{x_n}\right)^3\right], \tag{6.100}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\right) - \frac{1}{x_n^3}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right)\right], \tag{6.101}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\right) - \frac{1}{x_n^3}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right)\right], \tag{6.102}$$

$$\sigma_{nM} = -\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\left[c_{1M} - c_{2M} + 2(c_{1M} + 2c_{2M})\left(\frac{x_M}{x_n}\right)^3\right], \tag{6.103}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\left[c_{1M} - c_{2M} - (c_{1M} + 2\,c_{2M})\left(\frac{x_M}{x_n}\right)^3\right], \tag{6.104}$$

$$w_M = \kappa_{1M} + \frac{\kappa_{3M}}{x_n^6} + \frac{\kappa_{5M}}{x_n^3}, \tag{6.105}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) + \kappa_{5M}\ln\left(\frac{x_M}{x_E}\right)\right]d\varphi\, d\nu, \tag{6.106}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i = 1,2,3)$ are given by Equations (1.21), (1.22) and

(2.13), (2.18), respectively, and ζ_M, κ_{iM} ($i=3,5$; see Equation (6.12)) have the forms

$$\zeta_M = 1 - \left(\frac{x_M}{x_E}\right)^3,$$
$$\kappa_{3M} = 3\left(c_{1M} + 2\,c_{2M}\right)\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right)^2 + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right)\right]^2\right\},$$
$$\kappa_{5M} = -\frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right) - \frac{2\,\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right). \tag{6.107}$$

The coefficient κ_{1M} is given by Equation (6.98), where ζ_M in Equation (6.98) is given by Equation (6.107). The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{2M} \neq 0$, $C_{3M} \neq 0$, $C_{1M} = 0$. With regard to Equations (2.30), (2.31), (6.4)–(6.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\left[c_{3M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} - 2\left(\frac{x_M}{x_n}\right)^3\right], \tag{6.108}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\left[\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} - \left(\frac{x_M}{x_n}\right)^3\right], \tag{6.109}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M\,x_M^{c_{3M}-1}}\right)x_n^{c_{3M}-1} - \frac{1}{x_n^3}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right)\right], \tag{6.110}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M\,x_M^{c_{3M}-1}}\right)x_n^{c_{3M}-1} - \frac{1}{x_n^3}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right)\right], \tag{6.111}$$

$$\sigma_{nM} = -\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\left\{\left[c_{3M}\left(c_{1M}+c_{2M}\right)-2c_{2M}\right]\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right.$$
$$\left. -\,2\left(c_{1M}+2c_{2M}\right)\left(\frac{x_M}{x_n}\right)^3\right\}, \tag{6.112}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\left[\left(c_{1M}-c_{2M}c_{3M}\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}+\left(c_{1M}+2c_{2M}\right)\left(\frac{x_M}{x_n}\right)^3\right], \tag{6.113}$$

$$w_M = \kappa_{2M}\,x_n^{2(c_{3M}-1)}+\frac{\kappa_{3M}}{x_n^6}+\kappa_{6M}\,x_n^{c_{3M}-4}, \tag{6.114}$$

$$W_M = 4\int\limits_0^{\pi/2}\int\limits_0^{\pi/2}\left[\frac{\kappa_{2M}}{2c_{3M}+1}\left(x_M^{2c_{3M}+1}-x_E^{2c_{3M}+1}\right)+\frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3}-\frac{1}{x_M^3}\right)\right.$$
$$\left.+\,\frac{\kappa_{6M}}{c_{3M}-1}\left(x_M^{c_{3M}-1}-x_E^{c_{3M}-1}\right)\right]d\varphi\,d\nu, \tag{6.115}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i=1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_M, κ_{6M} (see Equation (6.12)) have the forms

$$\zeta_M = \left(\frac{x_E}{x_M}\right)^{c_{3M}-1}-\left(\frac{x_M}{x_E}\right)^3,$$
$$\kappa_{6M} = -\frac{x_M^3\left[2c_{2M}\left(1-c_{3M}\right)-c_{1M}\right]}{x_M^{c_{3M}-1}}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\right)^2$$
$$-\,\frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M\,x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right)$$
$$-\,\frac{2\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M\,x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}\,x_M^3}{\zeta_M}\right). \tag{6.116}$$

The coefficients κ_{2M}, κ_{3M} are given by Equations (6.21), (6.31), respectively, where ζ_M in Equations (6.21), (6.31) is given by Equation (6.116). The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{2M} \neq 0$, $C_{3M} \neq 0$. With regard to Equations (2.30), (2.31), (6.4)–(6.11), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}\left\{1 - \frac{1}{c_{3M}+2}\left[3c_{3M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} - 2\left(c_{3M}-1\right)\left(\frac{x_M}{x_n}\right)^3\right]\right\}, \tag{6.117}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}\left\{1 - \frac{1}{c_{3M}+2}\left[3\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} + \left(c_{3M}-1\right)\left(\frac{x_M}{x_n}\right)^3\right]\right\}, \tag{6.118}$$

$$\begin{aligned}\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -&\left\{\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)\right.\\ &\left.-\frac{1}{c_{3M}+2}\left[3\,\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M\,x_M^{c_{3M}-1}}\right)x_n^{c_{3M}-1} + \frac{c_{3M}-1}{x_n^3}\,\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}\,x_M^3}{\zeta_M}\right)\right]\right\},\end{aligned} \tag{6.119}$$

$$\begin{aligned}\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta&\left\{\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)\right.\\ &\left.-\frac{1}{c_{3M}+2}\left[3\,\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M\,x_M^{c_{3M}-1}}\right)x_n^{c_{3M}-1} + \frac{c_{3M}-1}{x_n^3}\,\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}\,x_M^3}{\zeta_M}\right)\right]\right\},\end{aligned} \tag{6.120}$$

$$\begin{aligned}\sigma_{nM} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}&\left\{c_{1M} - c_{2M} - \frac{3}{c_{3M}+2}\left[c_{3M}\left(c_{1M}+c_{2M}\right) - 2c_{2M}\right]\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right.\\ &\left.+\frac{2\left(c_{1M}+2c_{2M}\right)}{c_{3M}+2}\left(\frac{x_M}{x_n}\right)^3\right\},\end{aligned} \tag{6.121}$$

$$\begin{aligned}\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}&\left\{c_{1M} - c_{2M} - \frac{3\left(c_{1M}-c_{2M}c_{3M}\right)}{c_{3M}+2}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right.\\ &\left.-\frac{c_{1M}+2c_{2M}}{c_{3M}+2}\left(\frac{x_M}{x_n}\right)^3\right\},\end{aligned} \tag{6.122}$$

$$w_M = \kappa_{1M} + \kappa_{2M}\, x_n^{2(c_{3M}-1)} + \frac{\kappa_{3M}}{x_n^6} + \kappa_{4M}\, x_n^{c_{3M}-1} + \frac{\kappa_{5M}}{x_n^3} + \kappa_{6M}\, x_n^{c_{3M}-4}, \qquad (6.123)$$

$$\begin{aligned} W_M = 4 \int_0^{\pi/2}\int_0^{\pi/2} & \left[\frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \frac{\kappa_{2M}}{2\,c_{3M}+1}\left(x_M^{2\,c_{3M}+1} - x_E^{2\,c_{3M}+1}\right) \right. \\ & + \frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) + \frac{\kappa_{4M}}{c_{3M}+2}\left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2}\right) \\ & \left. + \kappa_{5M} \ln\left(\frac{x_M}{x_E}\right) + \frac{\kappa_{6M}}{c_{3M}-1}\left(x_M^{c_{3M}-1} - x_E^{c_{3M}-1}\right) \right] d\varphi\, d\nu, \qquad (6.124) \end{aligned}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i=1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_M, κ_{iM} ($i=2\ldots,6$; see Equation (6.12)) have the forms

$$\begin{aligned} \zeta_M = {} & c_{1M} - c_{2M} + \frac{1}{c_{3M}+2}\left(\frac{x_M}{x_E}\right)^3 \left\{ 2\,(c_{3M}-1)\,(c_{1M} + 2c_{Mm}) \right. \\ & \left. -3\left[c_{3M}\,(c_{1M}+c_{2M}) - 2\,c_{2M}\right]\left(\frac{x_E}{x_M}\right)^{c_{3M}+2} \right\}, \end{aligned}$$

$$\begin{aligned} \kappa_{2M} = {} & \left(\frac{3}{c_{3M}+2}\right)^2 \left(\left[\frac{(c_{1M}+c_{2M})\,c_{3M}^2}{2} + c_{1M} - 2\,c_{2M}\,c_{3M}\right] \left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M\, x_M^{c_{3M}-1}}\right)^2 \right. \\ & \left. + \frac{1}{s_{44M}} \left\{ \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2 \right\} \right), \end{aligned}$$

$$\begin{aligned} \kappa_{3M} = {} & \left(\frac{c_{3M}-1}{c_{3M}+2}\right)^2 \left(3\,(c_{1M}+2\,c_{2M}) \left(\frac{p_{1n}\,\rho_{2ME}\, x_M^3}{\zeta_M}\right)^2 \right. \\ & \left. + \frac{1}{s_{44M}} \left\{ \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}\, x_M^3}{\zeta_M}\right)\right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}\, x_M^3}{\zeta_M}\right)\right]^2 \right\} \right), \end{aligned}$$

$$\begin{aligned} \kappa_{4M} = {} & \frac{3\,(c_{2M}-c_{1M})}{x_M^{c_{3M}-1}} \left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\right)^2 \\ & - \frac{6}{s_{44M}\,(c_{3M}+2)} \left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\right) \frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M\, x_M^{c_{3M}-1}}\right) \right. \\ & \left. + \Theta^2 \frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M}\right) \frac{\partial}{\partial\nu}\left(\frac{p_{1n}\,\rho_{2ME}}{\zeta_M\, x_M^{c_{3M}-1}}\right) \right], \end{aligned}$$

$$\kappa_{5M} = \frac{2(1-c_{3M})}{s_{44M}(c_{3M}+2)} \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{1n}\rho_{2ME}}{\zeta_M} \right) \frac{\partial}{\partial \varphi} \left(\frac{p_{1n}\rho_{2ME} x_M^3}{\zeta_M} \right) \right.$$
$$\left. + \Theta^2 \frac{\partial}{\partial \nu} \left(\frac{p_{1n}\rho_{2ME}}{\zeta_M} \right) \frac{\partial}{\partial \nu} \left(\frac{p_{1n}\rho_{2ME} x_M^3}{\zeta_M} \right) \right],$$
$$\kappa_{6M} = \frac{3(c_{3M}-1)[2c_{2M}(1-c_{3M})-c_{1M}]}{x_M^{c_{3M}-4}} \left[\frac{p_{1n}\rho_{2ME}}{\zeta_M(c_{3M}+2)} \right]^2$$
$$+ \frac{6(c_{3M}-1)}{s_{44M}(c_{3M}+2)^2} \frac{\partial}{\partial \varphi} \left(\frac{p_{1n}\rho_{2ME}}{\zeta_M x_M^{c_{3M}-1}} \right) \frac{\partial}{\partial \varphi} \left(\frac{p_{1n}\rho_{2ME} x_M^3}{\zeta_M} \right)$$
$$+ \frac{6\Theta^2(c_{3M}-1)}{s_{44M}(c_{3M}+2)^2} \frac{\partial}{\partial \nu} \left(\frac{p_{1n}\rho_{2ME}}{\zeta_M x_M^{c_{3M}-1}} \right) \frac{\partial}{\partial \nu} \left(\frac{p_{1n}\rho_{2ME} x_M^3}{\zeta_M} \right). \tag{6.125}$$

The coefficient κ_{1M} is given by Equation (6.21), where ζ_M in Equation (6.21) is given by Equation (6.125). The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

6.3.2 Envelope

If $\beta_{IN} \neq \beta_E = \beta_M$, then $|u_{nE}| = f(x_n)$ represents a decreasing function of $x_n \in \langle x_{IN}, x_E \rangle$, and then the mathematical solutions, which are related to the integration constants C_{2E}, C_{3E} (see Equation (6.4)), are considered, i.e., $C_{1E} = 0$. With regard to one boundary condition (see Equation (2.33)), the thermal stresses in the ellipsoidal envelope are determined on the conditions $C_{2E} \neq 0$, $C_{3E} = 0$; $C_{3E} \neq 0$, $C_{2E} = 0$. If $C_{2E} \neq 0$, $C_{3E} = 0$, then the thermal strains and stresses in the ellipsoidal envelope are determined in Section 4.3.2 (see Equations (4.4), (6.4)). The normal stress p_{1n} in Section 4.3.2 is given by Equation (2.27), where ρ_{IN} and ρ_{1ME} in Equation (2.27) are given by Equations (6.89) and (6.135), respectively.

Condition $C_{3E} \neq 0, C_{2E} = 0$. With regard to Equations (2.33), (6.4)–(6.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\frac{2p_{1n}}{\zeta_E} \left(\frac{x_{IN}}{x_n} \right)^3, \tag{6.126}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = \frac{p_{1n}}{\zeta_E} \left(\frac{x_{IN}}{x_n} \right)^3, \tag{6.127}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi} = \frac{1}{\zeta_E x_n^3} \frac{\partial}{\partial \varphi} \left(p_{1n} x_{IN}^3 \right), \tag{6.128}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta} = \frac{\Theta}{\zeta_E\, x_n^3}\,\frac{\partial}{\partial \nu}\left(p_{1n}\, x_{IN}^3\right), \tag{6.129}$$

$$\sigma_{nE} = -p_{1n}\left(\frac{x_{IN}}{x_n}\right)^3, \tag{6.130}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = \frac{p_{1n}}{2}\left(\frac{x_{IN}}{x_n}\right)^3, \tag{6.131}$$

$$w_E = \frac{\kappa_{3E}}{x_n^6}, \tag{6.132}$$

$$W_E = \frac{4}{3}\int\limits_0^{\pi/2}\int\limits_0^{\pi/2} \kappa_{3E}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) d\varphi\, d\nu, \tag{6.133}$$

where Θ, x_E, x_M and s_{44E}, c_{iE} $(i=1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_E, κ_{3E} (see Equation (6.12)) have the forms

$$\begin{aligned}
&\zeta_E = 2\left(c_{1E} + 2c_{2E}\right),\\
&\kappa_{3E} = \frac{1}{\zeta_E^2}\Bigg(3\left(c_{1E} + 2\,c_{2E}\right)\left(p_{1n}\, x_{IN}^3\right)^2\\
&\qquad + \frac{1}{s_{44E}}\left\{\left[\frac{\partial}{\partial \varphi}\left(p_{1n}\, x_{IN}^3\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial \nu}\left(p_{1n}\, x_{IN}^3\right)\right]^2\right\}\Bigg).
\end{aligned} \tag{6.134}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} in Equation (2.27) is given by Equation (6.89). With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n = x_{IN}} = -p_{1n}\,\rho_{1ME}$, $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E} = -p_{1n}\,\rho_{2ME}$ (see Equation (6.127)), the coefficients ρ_{1ME}, ρ_{2ME} in Equations (2.27), (2.31) are derived as

$$\rho_{1ME} = -\frac{1}{\zeta_E}, \quad \rho_{2ME} = -\frac{1}{\zeta_E}\left(\frac{x_{IN}}{x_E}\right)^3, \tag{6.135}$$

6.4 Condition $\beta_{IN} = \beta_E \neq \beta_M$

If $\beta_{IN} = \beta_E \neq \beta_M$, then the thermal strains and stresses in the cell matrix are determined in Section 6.2.1. The radial stress p_{2n} in Section 6.2.1 is given by Equations (2.28). The coefficients ρ_M and ρ_{2INE} in Equations (2.26) are given by Equations (6.22), (6.32), (6.42), (6.52) and (6.143), respectively.

6.4.1 Envelope

If $\beta_{IN} = \beta_E \neq \beta_M$, then $|u_{nE}| = f(x_n)$ represents an increasing function of $x_n \in \langle x_{IN}, x_E \rangle$, and then $C_{1E} \neq 0$, $C_{2E} = C_{3E} = 0$ (see Equation (6.4)). With regard to Equations (2.34), (6.4)–(6.11), (2.22), (2.23), we get

$$\varepsilon_{nE} = \varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\frac{p_{2n}}{\zeta_E}, \tag{6.136}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\frac{1}{\zeta_E}\frac{\partial p_{2n}}{\partial \varphi}, \tag{6.137}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\frac{\Theta}{\zeta_E}\frac{\partial p_{2n}}{\partial \nu}, \tag{6.138}$$

$$\sigma_{nE} = \sigma_{\varphi E} = \sigma_{\theta E} = -p_{2n}, \tag{6.139}$$

$$w_E = \frac{1}{\zeta_E^2}\left(\frac{3\left(c_{1E} - c_{2E}\right)p_{2n}^2}{2} + \frac{1}{s_{44E}}\left[\left(\frac{\partial p_{2n}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial p_{2n}}{\partial \nu}\right)^2\right]\right), \tag{6.140}$$

$$W_E = \frac{4}{3}\int_0^{\pi/2}\int_0^{\pi/2} w_E\left(x_M^3 - x_E^3\right)d\varphi\, d\nu, \tag{6.141}$$

where Θ, x_E, x_M and s_{44E}, c_{iE} $(i = 1, 2)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_E has the form

$$\zeta_E = c_{1E} - c_{2E}. \tag{6.142}$$

The normal stress p_{2n} is given by Equation (2.28), where ρ_M in Equation (2.28) is given by Equations (3.24), (4.24), (4.34), (4.44), (4.54), (5.23), (5.33), (5.43), (5.53), (6.22), (6.32), (6.42), (6.52), (7.23), (7.33), (7.43), (7.53), (7.63), (7.73) with respect to minimum W_c (see Equation (2.23)). With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n = x_{IN}} = -p_{2n}\,\rho_{1INE}$, $\left(\varepsilon_{\varphi E}\right)_{x_n = x_E} = -p_{2n}\,\rho_{2INE}$ (see Equation (5.121)), the coefficients ρ_{1INE}, ρ_{2INE} in Equations (2.28), (2.37) are derived as

$$\rho_{1INE} = \rho_{2INE} = \frac{1}{\zeta_E}. \tag{6.143}$$

6.4.2 Inclusion

In case of the ellipsoidal inclusion, we get $C_{2IN} = C_{3IN} = 0$, otherwise we get $\left(u_{nIN}\right)_{x_n \to 0} \longrightarrow \pm\,\infty$, $\left(\varepsilon_{IN}\right)_{x_n \longrightarrow 0} \longrightarrow \pm\,\infty$, $\left(\sigma_{IN}\right)_{r \to 0} \longrightarrow \pm\infty$ due to $c_3 < 0$ (see Equations (2.18), (6.4)–(6.10)). With regard to Equations (2.35), (2.37), (6.4)–(6.11), (2.21), (2.22), (2.23), we get [2]–[23]

$$\varepsilon_{nIN} = \varepsilon_{\varphi IN} = \varepsilon_{\theta IN} = -p_{2n}\,\rho_{1INE}, \tag{6.144}$$

$$\varepsilon_{n\varphi IN} = s_{44IN}\,\sigma_{n\varphi IN} = -\,\frac{\partial\left(p_{2n}\,\rho_{1INE}\right)}{\partial\varphi}, \tag{6.145}$$

$$\varepsilon_{n\theta IN} = s_{44IN}\,\sigma_{n\theta IN} = -\Theta\,\frac{\partial\left(p_{2n}\,\rho_{1INE}\right)}{\partial\nu}, \tag{6.146}$$

$$\sigma_{nIN} = \sigma_{\varphi IN} = \sigma_{\theta IN} = -\,\frac{p_{2n}\,\rho_{1INE}}{\rho_{IN}}, \tag{6.147}$$

$$w_{IN} = \frac{3\left(p_{2n}\,\rho_{1INE}\right)^2}{\rho_{IN}} + \frac{2}{s_{44IN}}\left[\left(\frac{\partial\left(p_{2n}\,\rho_{1INE}\right)}{\partial\varphi}\right)^2 + \Theta\left(\frac{\partial\left(p_{2n}\,\rho_{1INE}\right)}{\partial\nu}\right)^2\right], \tag{6.148}$$

$$W_{IN} = \frac{4}{3}\int\limits_{0}^{\pi/2}\int\limits_{0}^{\pi/2} w_{IN}\,x_{IN}^3\,d\varphi\,d\nu, \tag{6.149}$$

where Θ, x_{IN}, s_{44IN}, ρ_{IN}, ρ_{1INE} are given by Equations (1.21), (1.22), (2.13), (6.89), (6.143). The normal stress p_{2n} is given by Equation (2.28). The coefficients ρ_M and ρ_{2INE} in Equation (2.28) are given by Equations (3.24), (4.24), (4.34), (4.44), (4.54), (5.23), (5.33), (5.43), (5.53), (6.22), (6.32), (6.42), (6.52), (7.23), (7.33), (7.43), (7.53), (7.63), (7.73) and (3.63), (4.140), (5.129), (5.139) (6.143), respectively, with respect to minimum W_c (see Equation (2.23)).

Chapter 7

Mathematical Model 5

7.1 Mathematical Procedure

Let the mathematical procedures ∂Eq. $(2.20)\,/\partial r$, Eq. $(6.2)\,/r$ be performed, and then we get

$$x_n\,\frac{\partial^2 U_n}{\partial x_n^2} + (1 - c_3)\,\frac{\partial U_n}{\partial x_n} = 0, \tag{7.1}$$

$$\frac{\partial U_n}{\partial x_n} = -s_{44}\,(c_1 + c_2)\left(x_n^2\,\frac{\partial^3 u_n}{\partial x_n^3} + 4x_n\,\frac{\partial^2 u_n}{\partial x_n^2}\right), \tag{7.2}$$

where s_{44} and c_1, c_2, $c_3 < 0$ are given by Equations (2.13) and (2.18), respectively. Let the mathematical procedure ∂Eq. $(7.2)\,/\partial r$ be performed, and then we get

$$\frac{\partial^2 U_n}{\partial x_n^2} = -s_{44}\,(c_1 + c_2)\left(x_n^2\,\frac{\partial^4 u_n}{\partial x_n^4} + 6x_n\,\frac{\partial^3 u_n}{\partial x_n^3} + 4\frac{\partial^2 u_n}{\partial x_n^2}\right). \tag{7.3}$$

Let Equations (6.2), (6.3) be substituted to (7.1), and then we get

$$x_n^2\,\frac{\partial^4 u_n}{\partial x_n^4} + (7 - c_3)\,x_n\,\frac{\partial^3 u_n}{\partial x_n^3} + 4\,(2 - c_3)\,\frac{\partial^2 u_n}{\partial x_n^2} = 0. \tag{7.4}$$

Let u_n be assumed in the form $u_n = x_n^{\lambda}$, then we get

$$u_n = C_1\,x_n + C_2\,x_n^{c_3} + \frac{C_3}{x_n^2} + C_4, \tag{7.5}$$

where $C_1 \ldots, C_4$ are integration constants, which are determined by the boundary conditions in Section 2.4. With regard to Equations (2.1)–(2.4), (2.14)–(2.17), (2.22), (7.6), we get

$$\varepsilon_n = C_1 + C_2\,c_3\,x_n^{c_3 - 1} - \frac{2\,C_3}{x_n^3}, \tag{7.6}$$

$$\varepsilon_\varphi = \varepsilon_\theta = C_1 + C_2\,x_n^{c_3 - 1} + \frac{C_3}{x_n^3} + \frac{C_4}{x_n}, \tag{7.7}$$

$$\varepsilon_{n\varphi} = s_{44}\,\sigma_{n\varphi} = \frac{\partial C_1}{\partial \varphi} + x_n^{c_3 - 1}\,\frac{\partial C_2}{\partial \varphi} + \frac{1}{x_n^3}\,\frac{\partial C_3}{\partial \varphi} + \frac{1}{x_n}\,\frac{\partial C_4}{\partial \varphi}, \tag{7.8}$$

$$\varepsilon_{n\theta} = s_{44}\,\sigma_{n\theta} = \Theta\left[\frac{\partial C_1}{\partial \nu} + x_n^{c_3-1}\,\frac{\partial C_2}{\partial \nu} + \frac{1}{x_n^3}\,\frac{\partial C_3}{\partial \nu} + \frac{1}{x_n}\,\frac{\partial C_4}{\partial \nu}\right], \tag{7.9}$$

$$\sigma_n = C_1\,(c_1 - c_2) + C_2\,[(c_1 + c_2)\,c_3 - 2\,c_2]\,x_n^{c_3-1} - \frac{2\,C_3\,(c_1 + 2\,c_2)}{x_n^3} - \frac{2\,c_2\,C_4}{x_n}, \tag{7.10}$$

$$\sigma_\varphi = \sigma_\theta = C_1\,(c_1 - c_2) + C_2\,(c_1 - c_2\,c_3)\,x_n^{c_3-1} + \frac{C_3\,(c_1 + 2\,c_2)}{x_n^3} + \frac{c_1\,C_4}{x_n}, \tag{7.11}$$

$$w = \kappa_1 + \kappa_2\,x_n^{2(c_3-1)} + \frac{\kappa_3}{x_n^6} + \frac{\kappa_4}{x_n^2} + (\kappa_5 + \kappa_9)\,x_n^{c_3-1} + \frac{\kappa_6}{x_n^3} + \kappa_7\,x_n^{c_3-4} + \frac{\kappa_8}{x_n} + \frac{\kappa_{10}}{x_n^4}, \tag{7.12}$$

where Θ, s_{44} are given by Equations (1.21), (2.13), respectively, and κ_i $(i = 4,5,6)$ is derived as

$$\begin{aligned}
\kappa_4 &= c_1\,C_4^2 + \frac{1}{s_{44}}\left[\left(\frac{\partial C_4}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial C_4}{\partial \nu}\right)^2\right], \\
\kappa_5 &= (c_1 - c_2)\,(2 + c_3)\,C_1\,C_2 + \frac{2}{s_{44}}\left(\frac{\partial C_1}{\partial \varphi}\,\frac{\partial C_2}{\partial \varphi} + \Theta^2\,\frac{\partial C_1}{\partial \nu}\,\frac{\partial C_2}{\partial \nu}\right), \\
\kappa_6 &= \frac{2}{s_{44}}\left(\frac{\partial C_1}{\partial \varphi}\,\frac{\partial C_3}{\partial \varphi} + \Theta^2\,\frac{\partial C_1}{\partial \nu}\,\frac{\partial C_3}{\partial \nu}\right), \\
\kappa_7 &= [2\,c_2\,(1 - c_3) - c_1]\,C_2\,C_3 + \frac{2}{s_{44}}\left(\frac{\partial C_2}{\partial \varphi}\,\frac{\partial C_3}{\partial \varphi} + \Theta^2\,\frac{\partial C_2}{\partial \nu}\,\frac{\partial C_3}{\partial \nu}\right), \\
\kappa_8 &= (c_1 - c_2)\,C_1\,C_4 + \frac{1}{s_{44}}\left(\frac{\partial C_1}{\partial \varphi}\,\frac{\partial C_4}{\partial \varphi} + \Theta^2\,\frac{\partial C_1}{\partial \nu}\,\frac{\partial C_4}{\partial \nu}\right), \\
\kappa_9 &= (c_1 - c_2\,c_3)\,C_2\,C_4 + \frac{1}{s_{44}}\left(\frac{\partial C_2}{\partial \varphi}\,\frac{\partial C_4}{\partial \varphi} + \Theta^2\,\frac{\partial C_2}{\partial \nu}\,\frac{\partial C_4}{\partial \nu}\right), \\
\kappa_{10} &= (c_1 + 2\,c_2)\,C_3\,C_4 + \frac{1}{s_{44}}\left(\frac{\partial C_3}{\partial \varphi}\,\frac{\partial C_4}{\partial \varphi} + \Theta^2\,\frac{\partial C_3}{\partial \nu}\,\frac{\partial C_4}{\partial \nu}\right).
\end{aligned} \tag{7.13}$$

The coefficient κ_i $(i = 1,2,3)$ is given by Equation (6.13). In case of the ellipsoidal inclusion, we get $C_{2IN} = C_{3IN} = C_{4IN} = 0$, otherwise we get $(u_{nIN})_{x_n \to 0} \longrightarrow \pm\,\infty$, $(\varepsilon_{IN})_{x_n \longrightarrow 0} \longrightarrow \pm\,\infty$, $(\sigma_{IN})_{r \to 0} \longrightarrow \pm\infty$ due to $c_3 < 0$ (see Equations (2.18), (6.4)–(6.10)). In case of $C_{1IN} \neq 0$ (see Equations (6.4), (7.5)), the mathematical solutions for the ellipsoidal inclusion is presented in Section 6.3.

7.2 Condition $\beta_{IN} \neq \beta_E \neq \beta_M$

7.2.1 Matrix

The integration constants $C_{1M}, C_{2M}, C_{3M}, C_{4M}$ for the matrix (see Equation (7.5)) are determined by the boundary conditions (2.29), (2.30) or (2.29)–(2.32). The boundary conditions result in the following combinations of C_{1M}, C_{2M}, C_{3M}, C_{4M}, where the combinations of C_{1M}, C_{2M}, C_{3M} are presented in Section 6.2.1. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{4M} \neq 0$, $C_{2M} = C_{3M} = 0$. With regard to Equations (2.29), (2.30), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M}, \tag{7.14}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left(1 - \frac{1}{x_n}\right), \tag{7.15}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left(1 - \frac{1}{x_n}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right), \tag{7.16}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left(1 - \frac{1}{x_n}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right), \tag{7.17}$$

$$\sigma_{nM} = -\frac{p_{2n}}{\zeta_M}\left(c_{1M} - c_{2M} + \frac{2c_{Mm}}{x_n}\right), \tag{7.18}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left(c_{1M} - c_{2M} - \frac{c_{1M}}{x_n}\right), \tag{7.19}$$

$$w_M = \kappa_{1M} + \frac{\kappa_{4M}}{x_n^2} + \frac{\kappa_{8M}}{x_n}, \tag{7.20}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \kappa_{4M}\left(x_M - x_E\right) + \frac{\kappa_{8M}}{2}\left(x_M^2 - x_E^2\right)\right] d\varphi\, d\nu, \tag{7.21}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} ($i = 1,2$) are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_M, κ_{jM} ($j = 4,8$; see Equation (7.13)) have the forms

$$\zeta_M = c_{1M} - c_{2M} + \frac{2c_{2M}x_M}{x_E},$$
$$\kappa_{4M} = c_{1M}\left(\frac{p_{2n}}{\zeta_M}\right)^2 + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 + \left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2\right\},$$
$$\kappa_{8m} = (c_{2M} - c_{1M})\left(\frac{p_{2n}}{\zeta_M}\right)^2 - \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2 + \left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right]^2\right\}. \quad (7.22)$$

The coefficient κ_{1M} is given by Equation (6.21), where ζ_M in Equation (6.21) is given by Equation (7.22). The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i, j = 1{,}2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to Equation (7.15), the coefficient ρ_M in Equation ((2.26)) is derived as

$$\rho_M = \frac{1}{\zeta_M}\left(1 - \frac{1}{x_E}\right). \quad (7.23)$$

Conditions $C_{2M} \neq 0$, $C_{4M} \neq 0$, $C_{1M} = C_{3M} = 0$. With regard to Equations (2.29), (2.30), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}c_{3M}}{\zeta_M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}, \quad (7.24)$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} - \frac{1}{x_n}\right], \quad (7.25)$$

$$\varepsilon_{n\varphi M} = s_{44M}\sigma_{n\varphi M} = -\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\kappa x_M^{c_{3M}-1}}\right) - \frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\right], \quad (7.26)$$

$$\varepsilon_{n\theta M} = s_{44M}\sigma_{n\theta M} = -\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\kappa x_M^{c_{3M}-1}}\right) - \frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\right], \quad (7.27)$$

$$\sigma_{nM} = -\frac{p_{2n}}{\zeta_M}\left\{\left[c_{3M}(c_{1M} + c_{2M}) - 2c_{2M}\right]\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} + \frac{2c_{2M}}{x_n}\right\}, \quad (7.28)$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[(c_{1M} - c_{2M}c_{3M})\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} - \frac{c_{1M}}{x_n}\right], \quad (7.29)$$

$$w_M = \kappa_{2M}\, x_n^{2(c_{3M}-1)} + \frac{\kappa_{4M}}{x_n^2} + \kappa_{9M}\, x_n^{c_{3M}-1}, \tag{7.30}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{2M}}{2c_{3M}+1}\left(x_M^{2c_{3M}+1} - x_E^{2c_{3M}+1}\right) + \kappa_{4M}\,(x_M - x_E)\right.$$
$$\left. + \frac{\kappa_{9M}}{c_{3M}+2}\left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2}\right)\right] d\varphi\, d\nu, \tag{7.31}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} ($i = 1,2,3$) are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_M, κ_{2M} (see Equation (6.13)), κ_{9M} (see Equation (7.13)) have the forms

$$\zeta_M = \left[c_{3M}\,(c_{1M}+c_{2M}) - 2c_{2M}\right]\left(\frac{x_E}{x_M}\right)^{c_{3M}-1} + \frac{2c_{2M}\, x_M}{x_E},$$
$$\kappa_{2M} = \left[\frac{c_{3M}^2\,(c_{1M}+c_{2M})}{2} + c_{1M} - 2c_{2M}c_{3M}\right]\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)^2$$
$$+ \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2\right\},$$
$$\kappa_{9M} = -\frac{c_{1M} - c_{2M}c_{3M}}{x_M^{c_{3M}-1}}\left(\frac{p_{2n}}{\zeta_M}\right)^2 - \frac{1}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)$$
$$- \frac{\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right). \tag{7.32}$$

The coefficient κ_{4M} is given by Equations (6.21), where ζ_M in Equations (6.21) is given by Equation (7.32). The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} ($i, j = 1,2$) in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to Equation (7.25), the coefficient ρ_M in Equation ((2.26)) is derived as

$$\rho_M = \frac{1}{\zeta_M}\left[\left(\frac{x_E}{x_M}\right)^{c_{3M}-1} - \frac{1}{x_E}\right]. \tag{7.33}$$

Conditions $C_{3M} \neq 0$, $C_{4M} \neq 0$, $C_{1M} = C_{2M} = 0$. With regard to Equations (2.29), (2.30), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = \frac{2p_{2n}}{\zeta_M}\left(\frac{x_E}{x_n}\right)^3, \tag{7.34}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\left(\frac{x_E}{x_n}\right)^3 - \frac{1}{x_n}\right], \tag{7.35}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left[\left(\frac{x_E}{x_n}\right)^3 - \frac{1}{x_n}\right]\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right), \tag{7.36}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\left[\left(\frac{x_E}{x_n}\right)^3 - \frac{1}{x_n}\right]\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right), \tag{7.37}$$

$$\sigma_{nM} = \frac{2p_{2n}}{\zeta_M}\left[(c_{1M} + 2c_{2M})\left(\frac{x_E}{x_n}\right)^3 - \frac{c_{2M}}{x_n}\right], \tag{7.38}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[(c_{1M} - 2c_{2M})\left(\frac{x_E}{x_n}\right)^3 - \frac{c_{1M}}{x_n}\right], \tag{7.39}$$

$$w_M = \frac{\kappa_{3M}}{x_n^6} + \frac{\kappa_{4M}}{x_n^2} + \frac{\kappa_{10M}}{x_n^4}, \tag{7.40}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) + \kappa_{4M}(x_M - x_E) + \kappa_{10M}\left(\frac{1}{x_E} - \frac{1}{x_M}\right)\right] d\varphi\, d\nu, \tag{7.41}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i = 1,2)$ are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_M, κ_{3M} (see Equation (6.13)), κ_{10M} (see Equation (7.13)) have the forms

$$\zeta_M = -\left[2(c_{1M} + 2c_{2M}) + 2c_{2M}\left(\frac{x_E}{x_M}\right)^2\right],$$

$$\kappa_{3M} = 3(c_{1M} + 2c_{2M})\left(\frac{p_{2n}x_E^3}{\zeta_M}\right)^2 + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_E^3}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_E^3}{\zeta_M}\right)\right]^2\right\},$$

$$\kappa_{10M} = -x_E^3 \left(c_{1M} + 2\,c_{2M}\right) \left(\frac{p_{2n}}{\zeta_M}\right)^2 - \frac{1}{s_{44M}} \left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\, x_E^3}{\zeta_M}\right) \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}}{\zeta_M}\right)\right] - \frac{\Theta^2}{s_{44M}} \left[\frac{\partial}{\partial \nu} \left(\frac{p_{2n}\, x_E^3}{\zeta_M}\right) \frac{\partial}{\partial \nu} \left(\frac{p_{2n}}{\zeta_M}\right)\right]. \tag{7.42}$$

The coefficient κ_{4M} is given by Equations (6.21), where ζ_M in Equations (6.21) is given by Equation (7.42). The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i, j = 1{,}2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to Equation (7.35), the coefficient ρ_M in Equation ((2.26)) is derived as

$$\rho_M = \frac{x_E - 1}{\zeta_M\, x_E}. \tag{7.43}$$

Conditions $C_{1M} \neq 0, C_{2M} \neq 0, C_{4M} \neq 0, C_{3M} = 0$. With regard to Equations (2.29)–(2.32), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M} \left[1 - \left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right], \tag{7.44}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M} \left\{1 - \frac{1}{c_{3M}} \left[\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} + \frac{(c_{3M} - 1)\, x_M}{x_n}\right]\right\}, \tag{7.45}$$

$$\varepsilon_{n\varphi M} = s_{44M}\, \sigma_{n\varphi M} = -\left\{\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}}{\zeta_M}\right) - \frac{1}{c_{3M}} \left[x_n^{c_{3M}-1} \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right) + \frac{c_{3M} - 1}{x_n} \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\, x_M}{\zeta_M}\right)\right]\right\}, \tag{7.46}$$

$$\varepsilon_{n\theta M} = s_{44M}\, \sigma_{n\theta M} = -\left\{\frac{\partial}{\partial \nu} \left(\frac{p_{2n}}{\zeta_M}\right) - \frac{1}{c_{3M}} \left[x_n^{c_{3M}-1} \frac{\partial}{\partial \nu} \left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right) + \frac{c_{3M} - 1}{x_n} \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\, x_M}{\zeta_M}\right)\right]\right\}, \tag{7.47}$$

$$\sigma_{nM} = -\frac{p_{2n}}{\zeta_M} \left(c_{1M} - c_{2M} - \frac{1}{c_{3M}} \left\{\left[c_{3M} \left(c_{1M} + c_{2M}\right) - 2c_{2M}\right] \left(\frac{x_n}{x_M}\right)^{c_{3M}-1} + \frac{2c_{2M}\, (c_{3M} - 1)\, x_M}{c_{3M}\, x_n}\right\}\right), \tag{7.48}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left\{c_{1M} - c_{2M} - \frac{1}{c_{3M}}\left[(c_{1M} - c_{2M}c_{3M})\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} + \frac{c_{1M}(c_{1M}-1)\,x_M}{x_n}\right]\right\}, \tag{7.49}$$

$$w_M = \kappa_{1M} + \kappa_{2M}\,x_n^{2(c_{3M}-1)} + \frac{\kappa_{4M}}{x_n^2} + (\kappa_{5M} + \kappa_{9M})\,x_n^{c_{3M}-1} + \frac{\kappa_{8M}}{x_n}, \tag{7.50}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \frac{\kappa_{2M}}{2c_{3M}+1}\left(x_M^{2c_{3M}+1} - x_E^{2c_{3M}+1}\right) + \kappa_{4M}(x_M - x_E) + \frac{\kappa_{5M}+\kappa_{9M}}{c_{3M}+2}\left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2}\right) + \frac{\kappa_{8M}}{2}\left(x_M^2 - x_E^2\right)\right] d\varphi\, d\nu, \tag{7.51}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i=1,2,3)$ are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_M, κ_{iM} ($i=1,2$; see Equation (6.13)), κ_{jM} ($j=4,5,8,9$; see Equation (7.13)) have the forms

$$\zeta_M = (c_{1M} - c_{2M}) - \frac{[c_{3M}(c_{1M}+c_{2M}) - 2c_{2M}]}{c_{3M}}\left(\frac{x_E}{x_M}\right)^{c_{3M}-1} + \frac{2c_{2M}(c_{3M}-1)\,x_M}{c_{3M}\,x_E},$$

$$\kappa_{4M} = c_{1M}\left[\frac{p_{2n}x_M(c_{3M}-1)}{\zeta_M\,c_{3M}}\right]^2 + \frac{1}{s_{44M}}\left\{\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}x_M(c_{3M}-1)}{\zeta_M\,c_{3M}}\right]\right\}^2 + \frac{\Theta^2}{s_{44M}}\left\{\frac{\partial}{\partial\nu}\left[\frac{p_{2n}x_M(c_{3M}-1)}{\zeta_M\,c_{3M}}\right]\right\}^2,$$

$$\kappa_{5M} = -\frac{(c_{1M}-c_{2M})(2+c_{3M})}{c_{3M}\,x_M^{c_{3M}-1}}\left(\frac{p_{2n}}{\zeta_M}\right)^2 - \frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\,c_{3M}\,x_M^{c_{3M}-1}}\right) - \frac{2\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\,c_{3M}\,x_M^{c_{3M}-1}}\right),$$

$$\kappa_{8M} = -(c_{1M}-c_{2M})\left(\frac{p_{2n}}{\zeta_M}\right)\left[\frac{p_{2n}x_M(c_{3M}-1)}{\zeta_M\,c_{3M}}\right] - \frac{1}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left[\frac{p_{2n}x_M(c_{3M}-1)}{\zeta_M\,c_{3M}}\right] - \frac{\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left[\frac{p_{2n}x_M(c_{3M}-1)}{\zeta_M\,c_{3M}}\right],$$

$$\kappa_{9M} = \frac{(c_{1M} - c_{2M}\, c_{3M})\,(c_{3M} - 1)}{x_M^{c_{3M}-2}} \left(\frac{p_{2n}}{\zeta_M\, c_{3M}}\right)^2 + \frac{(c_{3M} - 1)}{s_{44M}\, c_{3M}^2} \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right) \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\, x_M}{\zeta_M}\right) + \frac{\Theta^2\,(c_{3M} - 1)}{s_{44M}\, c_{3M}^2} \frac{\partial}{\partial \nu} \left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right) \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\, x_M}{\zeta_M}\right). \tag{7.52}$$

The coefficient κ_{iM} $(i = 1{,}2)$ is given by Equations (6.21), where ζ_M in Equations (6.21) is given by Equation (7.52). The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i, j = 1{,}2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to Equation (7.45), the coefficient ρ_M in Equation ((2.26)) is derived as

$$\rho_M = \frac{1}{\zeta_M} \left\{ 1 - \frac{1}{c_{3M}} \left[\left(\frac{x_E}{x_M}\right)^{c_{3M}-1} + \frac{(c_{3M} - 1)\, x_M}{x_E} \right] \right\}. \tag{7.53}$$

Conditions $C_{1M} \neq 0, C_{3M} \neq 0, C_{4M} \neq 0, C_{2M} = 0$. With regard to Equations (2.29)–(2.32), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M} \left[1 - \frac{3}{2} \left(\frac{x_M}{x_n}\right)^3 \right], \tag{7.54}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M} \left[1 + \frac{1}{2} \left(\frac{x_M}{x_n}\right)^3 - \frac{3 x_M}{2 x_n} \right], \tag{7.55}$$

$$\varepsilon_{n\varphi M} = s_{44M}\, \sigma_{n\varphi M} = -\left[\frac{\partial}{\partial \varphi} \left(\frac{p_{2n}}{\zeta_M}\right) + \frac{1}{2 x_n^3} \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right) - \frac{3}{2 x_n} \frac{\partial}{\partial \varphi} \left(\frac{p_{2n}\, x_M}{\zeta_M}\right) \right], \tag{7.56}$$

$$\varepsilon_{n\theta M} = s_{44M}\, \sigma_{n\theta M} = -\left[\frac{\partial}{\partial \nu} \left(\frac{p_{2n}}{\zeta_M}\right) + \frac{1}{2 x_n^3} \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right) - \frac{3}{2 x_n} \frac{\partial}{\partial \nu} \left(\frac{p_{2n}\, x_M}{\zeta_M}\right) \right], \tag{7.57}$$

$$\sigma_{nM} = -\frac{p_{2n}}{\zeta_M} \left[c_{1M} - c_{2M} - (c_{1M} + 2 c_{2M}) \left(\frac{x_M}{x_n}\right)^3 + \frac{3 c_{2M}\, x_M}{x_n} \right], \tag{7.58}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[c_{1M} - c_{2M} + \frac{c_{1M} + 2c_{2M}}{2}\left(\frac{x_M}{x_n}\right)^3 - \frac{3c_{1M}x_M}{2x_n}\right], \tag{7.59}$$

$$w_M = \kappa_{1M} + \frac{\kappa_{3M}}{x_n^6} + \frac{\kappa_{4M}}{x_n^2} + \frac{\kappa_{6M}}{x_n^3} + \frac{\kappa_{8M}}{x_n} + \frac{\kappa_{10M}}{x_n^4}, \tag{7.60}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) + \kappa_{4M}\left(x_M - x_E\right)\right.$$
$$\left. + \kappa_{6M}\ln\left(\frac{x_M}{x_E}\right) + \frac{\kappa_{8M}}{2}\left(x_M^2 - x_E^2\right) + \kappa_{10M}\left(\frac{1}{x_E} - \frac{1}{x_M}\right)\right] d\varphi\, d\nu, \tag{7.61}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i=1,2)$ are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_M, κ_{3M} (see Equation (6.13)), κ_{iM} ($i=4,6,8,10$; see Equation (7.13)) have the forms

$$\zeta_M = (c_{1M} - c_{2M}) - (c_{1M} + 2c_{2M})\left(\frac{x_M}{x_E}\right)^3 + \frac{3c_{2M}x_M}{x_E},$$
$$\kappa_{3M} = 3\left(c_{1M} + 2c_{2M}\right)\left(\frac{p_{2n}x_M^3}{2\zeta_M}\right)^2$$
$$+ \frac{1}{s_{44}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M^3}{2\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M^3}{2\zeta_M}\right)\right]^2\right\},$$
$$\kappa_{4M} = c_{1M}\left(\frac{3p_{2n}x_M}{2\zeta_M}\right)^2 + \frac{1}{s_{44}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{3p_{2n}x_M}{2\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{3p_{2n}x_M}{2\zeta_M}\right)\right]^2\right\},$$
$$\kappa_{6M} = \frac{2}{s_{44}}\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M^3}{2\zeta_M}\right) + \Theta^2\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M^3}{2\zeta_M}\right)\right],$$
$$\kappa_{8M} = -\frac{3x_M\left(c_{1M} - c_{2M}\right)}{2}\left(\frac{p_{2n}}{\zeta_M}\right)^2$$
$$- \frac{3}{2s_{44}}\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M}{\zeta_M}\right) + \Theta^2\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M}{\zeta_M}\right)\right],$$
$$\kappa_{10M} = -\frac{3x_M^4\left(c_{1M} + 2c_{2M}\right)}{4}\left(\frac{p_{2n}}{\zeta_M}\right)^2$$
$$- \frac{3}{4s_{44}}\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}x_M}{\zeta_M}\right) + \Theta^2\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M^3}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}x_M}{\zeta_M}\right)\right]. \tag{7.62}$$

The coefficient κ_{1M} is given by Equations (6.21), where ζ_M in Equations (6.21) is given by Equation (7.62). The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i, j = 1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to Equation (7.55), the coefficient ρ_M in Equation ((2.26)) is derived as

$$\rho_M = \frac{1}{\zeta_M}\left[1 + \frac{1}{2}\left(\frac{x_M}{x_E}\right)^3 - \frac{3x_M}{2x_E}\right]. \tag{7.63}$$

Conditions $C_{2M} \neq 0, C_{3M} \neq 0, C_{4M} \neq 0, C_{1M} = 0$. With regard to Equations (2.29)–(2.32), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{2n}}{\zeta_M}\left[c_{3M}\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} - c_{3M}\left(\frac{x_M}{x_n}\right)^3\right], \tag{7.64}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{2n}}{\zeta_M}\left[\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} + \frac{c_{3M}}{2}\left(\frac{x_M}{x_n}\right)^3 - \frac{(c_{3M}+2)\,x_M}{2x_n}\right], \tag{7.65}$$

$$\begin{aligned}\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -&\left[x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right) + \frac{c_{3M}}{2x_n^3}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\,x_M^3}{\zeta_M}\right)\right.\\ &\left. + \frac{(c_{3M}+2)}{2x_n}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\,x_M}{\zeta_M}\right)\right],\end{aligned} \tag{7.66}$$

$$\begin{aligned}\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -&\left[x_n^{c_{3M}-1}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right) + \frac{c_{3M}}{2x_n^3}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\,x_M^3}{\zeta_M}\right)\right.\\ &\left. + \frac{(c_{3M}+2)}{2x_n}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\,x_M}{\zeta_M}\right)\right],\end{aligned} \tag{7.67}$$

$$\begin{aligned}\sigma_{nM} = -\frac{p_{2n}}{\zeta_M}&\left\{\left[c_{3M}\left(c_{1M}+c_{2M}\right) - 2c_{2M}\right]\left(\frac{x_n}{x_M}\right)^{c_{3M}-1}\right.\\ &\left. - \left(c_{3M}c_{1M}+2c_{2M}\right)\left(\frac{x_M}{x_n}\right)^3 + \frac{c_{2M}\left(c_{3M}+2\right)x_M}{x_n}\right\},\end{aligned} \tag{7.68}$$

$$\begin{aligned}\sigma_{\varphi M} = \sigma_{\theta M} = \frac{p_{2n}}{\zeta_M}&\left[\left(c_{1M}-c_{2M}c_{3M}\right)\left(\frac{x_n}{x_M}\right)^{c_{3M}-1} + \frac{c_{3M}\left(c_{1M}+2c_{2M}\right)}{2}\left(\frac{x_M}{x_n}\right)^3\right.\\ &\left. - \frac{c_{1M}\left(c_{3M}+2\right)x_M}{2x_n}\right],\end{aligned} \tag{7.69}$$

$$w_M = \kappa_{2M}\, x_n^{2(c_{3M}-1)} + \frac{\kappa_{3M}}{x_n^6} + \frac{\kappa_{4M}}{x_n^2} + \kappa_{7M}\, x_n^{c_{3M}-4} + \kappa_{9M} x_n^{c_{3M}-1} + \frac{\kappa_{10M}}{x_n^4}, \qquad (7.70)$$

$$\begin{aligned} W_M = 4 \int_0^{\pi/2}\int_0^{\pi/2} \Bigg[& \frac{\kappa_{2M}}{2\,c_{3M}+1}\left(x_M^{2\,c_{3M}+1} - x_E^{2\,c_{3M}+1}\right) + \frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) \\ & + \kappa_{4M}\,(x_M - x_E) + \frac{\kappa_{7M}}{c_{3M}-1}\left(x_M^{c_{3M}-1} - x_E^{c_{3M}-1}\right) \\ & + \frac{\kappa_{9M}}{c_{3M}+2}\left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2}\right) + \kappa_{10M}\left(\frac{1}{x_E} - \frac{1}{x_M}\right)\Bigg]\, d\varphi\, d\nu, \quad (7.71) \end{aligned}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i=1,2)$ are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_M, κ_{iM} ($i=2,3$; see Equation (6.13)), κ_{jM} ($j=4,7,9,10$; see Equation (7.13)) have the forms

$$\begin{aligned} \zeta_M = x_M^{c_{3M}-1} \Bigg\{ & \left[c_{3M}\,(c_{1M}+c_{2M}) - 2c_{2M}\right]\left(\frac{x_E}{x_M}\right)^{c_{3M}-1} \\ & - c_{3M}\,(c_{1M}+2c_{2M})\left(\frac{x_M}{x_E}\right)^3 + \frac{c_{2M}\,(c_{3M}+2)\,x_M}{x_E}\Bigg\}, \end{aligned}$$

$$\begin{aligned} \kappa_{2M} = & \left[\frac{(c_{1M}+c_{2M})\,c_{3M}^2}{2} + c_{1M} - 2\,c_{2M}\,c_{3M}\right]\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)^2 \\ & + \frac{1}{s_{44M}}\left\{\left[\frac{\partial C_2}{\partial \varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2 + \Theta^2\left[\frac{\partial C_2}{\partial \nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\right]^2\right\}, \end{aligned}$$

$$\begin{aligned} \kappa_{3M} = & \,3\,(c_{1M}+2\,c_{2M})\left(\frac{p_{2n}\,c_{3M}\,x_M^3}{2\,\zeta_M}\right)^2 \\ & + \frac{1}{2s_{44M}}\left\{\left[\frac{\partial}{\partial \varphi}\left(\frac{p_{2n}\,c_{3M}\,x_M^3}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial \nu}\left(\frac{p_{2n}\,c_{3M}\,x_M^3}{\zeta_M}\right)\right]^2\right\}, \end{aligned}$$

$$\begin{aligned} \kappa_{4M} = & \,c_{1M}\left[\frac{p_{2n}\,x_M\,(c_{3M}+2)}{2\,\zeta_M}\right]^2 \\ & + \frac{c_{3M}+2}{s_{44M}}\left\{\left[\frac{\partial}{\partial \varphi}\left(\frac{p_{2n}\,x_M}{2\,\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial \nu}\left(\frac{p_{2n}\,x_M}{2\,\zeta_M}\right)\right]^2\right\}, \end{aligned}$$

$$\kappa_{7M} = \left[2\,c_{2M}\,(1-c_{3M}) - c_{1M}\right]\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\left(\frac{p_{2n}\,c_{3M}\,x_M^3}{2\,\zeta_M}\right)$$

$$+\frac{c_{3M}}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right)$$

$$+\frac{\Theta^2 c_{3M}}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right),$$

$$\kappa_{9M}=-\frac{x_M\left(c_{1M}-c_{2M}\,c_{3M}\right)\left(c_{3M}+2\right)}{2x_M^{c_{3M}-1}}\left(\frac{p_{2n}}{\zeta_M}\right)^2$$

$$-\frac{c_{3M}+2}{2\,s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\, x_M}{\zeta_M}\right)$$

$$-\frac{\Theta^2\left(c_{3M}+2\right)}{2\,s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}}{\zeta_M\, x_M^{c_{3M}-1}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\, x_M}{\zeta_M}\right)$$

$$\kappa_{10M}=-\,c_{3M}\left(c_{1M}+2\,c_{2M}\right)\left(c_{3M}+2\right)\left(\frac{p_{2n}\, x_M^2}{2\,\zeta_M}\right)^2$$

$$-\frac{c_{3M}\left(c_{3M}+2\right)}{4\,s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{2n}\, x_M}{\zeta_M}\right)$$

$$-\frac{\Theta^2\, c_{3M}\left(c_{3M}+2\right)}{4\,s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\, x_M^3}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{2n}\, x_M}{\zeta_M}\right). \tag{7.72}$$

The normal stress p_{2n} is given by Equation (2.26), where ρ_{IN} and ρ_{ijE} $(i,j=1,2)$ in Equation (2.26) are given by Equations (6.89) and (6.62), (6.72), (6.82), (7.83), (7.93), (7.103), respectively, with respect to minimum W_c (see Equation (2.23)). With regard to Equation (7.65), the coefficient ρ_M in Equation ((2.26)) is derived as

$$\rho_M=\frac{1}{\zeta_M}\left[\left(\frac{x_E}{x_M}\right)^{c_{3M}-1}+\frac{c_{3M}}{2}\left(\frac{x_M}{x_E}\right)^3-\frac{\left(c_{3M}+2\right)x_M}{2x_E}\right]. \tag{7.73}$$

7.2.2 Envelope

The integration constants C_{1E}, C_{2E}, C_{3E} for the envelope (see Equation (6.4)) are determined by the boundary conditions (2.33), (2.34). The boundary conditions result in the following combinations of C_{1E}, C_{2E}, C_{3E}, i.e., $C_{1E}\neq 0$, $C_{2E}\neq 0$, $C_{3E}=0$; $C_{1E}\neq 0$, $C_{3E}\neq 0$, $C_{2E}=0$; $C_{2E}\neq 0$, $C_{3E}\neq 0$, $C_{1E}=0$. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1E}\neq 0$, $C_{4E}\neq 0$, $C_{2E}=C_{3E}=0$. With regard to Equations (2.33), (2.34), (7.5)–(7.12), (2.22), (2.23), we get

$$\varepsilon_{nE} = -\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right), \tag{7.74}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}} + \frac{p_{1n} - p_{2n}}{\zeta_{3E}\, x_n}\right), \tag{7.75}$$

$$\varepsilon_{n\varphi E} = s_{44E}\, \sigma_{n\varphi E} = -\left[\frac{\partial}{\partial \varphi}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{1}{x_n}\frac{\partial}{\partial \varphi}\left(\frac{p_{1n} - p_{2n}}{\zeta_{3E}}\right)\right], \tag{7.76}$$

$$\varepsilon_{n\theta E} = s_{44E}\, \sigma_{n\theta E} = -\Theta\left[\frac{\partial}{\partial \nu}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{1}{x_n}\frac{\partial}{\partial \nu}\left(\frac{p_{1n} - p_{2n}}{\zeta_{3E}}\right)\right], \tag{7.77}$$

$$\sigma_{nE} = -\left[(c_{1E} - c_{2E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) - \frac{2\, c_{2E}\,(p_{1n} - p_{2n})}{\zeta_{3E}\, x_n}\right], \tag{7.78}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\left[(c_{1E} - c_{2E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{c_{1E}\,(p_{1n} - p_{2n})}{\zeta_{3E}\, x_n}\right], \tag{7.79}$$

$$w_E = \kappa_{1E} + \frac{\kappa_{4E}}{x_n^2} + \frac{\kappa_{8E}}{x_n}, \tag{7.80}$$

$$W_E = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{1E}}{3}\left(x_E^3 - x_{IN}^3\right) + \kappa_{4E}\,(x_E - x_{IN}) + \frac{\kappa_{8E}}{2}\left(x_E^2 - x_{IN}^2\right)\right] d\varphi\, d\nu, \tag{7.81}$$

where Θ, x_{IN}, x_E and s_{44E}, c_{iE} are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_{iE} $(i = 1,2,3)$, κ_{jE} $(i = 4,8$; see Equation (7.13)) have the forms

$$\zeta_{1E} = (c_{1E} - c_{2E})\left(1 - \frac{x_E}{x_{IN}}\right), \quad \zeta_{2E} = (c_{1E} - c_{2E})\left(\frac{x_{IN}}{x_E} - 1\right),$$
$$\zeta_{3E} = \frac{2c_{2E}\,(x_{IN} - x_E)}{x_{IN}\, x_E},$$
$$\kappa_{4E} = c_{1E}\, C_{4E}^2 + \frac{1}{s_{44E}}\left[\left(\frac{\partial C_{4E}}{\partial \varphi}\right)^2 + \Theta^2\left(\frac{\partial C_{4E}}{\partial \nu}\right)^2\right],$$
$$\kappa_{8E} = (c_{1E} - c_{2E})\, C_{1E}\, C_{4E} + \frac{1}{s_{44E}}\left(\frac{\partial C_{1E}}{\partial \varphi}\frac{\partial C_{4E}}{\partial \varphi} + \Theta^2\frac{\partial C_{1E}}{\partial \nu}\frac{\partial C_{4E}}{\partial \nu}\right). \tag{7.82}$$

The coefficient κ_{1E} is given by Equation (6.61), where ζ_{iE} $(i = 1,2,3)$ in Equation (6.61) is given by Equation (7.82). The normal stresses p_{1n}, p_{2n} are given by

Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (7.23), (7.33), (7.43), (7.53), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}} = -\left(p_{1n}\,\rho_{11E} + p_{2n}\,\rho_{12E}\right)$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E} = -\left(p_{1n}\,\rho_{21E} + p_{2n}\,\rho_{22E}\right)$ and Equation (7.75), the coefficient ρ_{ijE} $(i, j = 1{,}2)$ in Equations (2.25), (2.26) is derived as

$$\rho_{1iE} = \left(\delta_{1i} - \delta_{2i}\right)\left(\frac{1}{\zeta_{iE}} + \frac{1}{\zeta_{3E}\,x_{IN}}\right), \quad \rho_{2iE} = \left(\delta_{1i} - \delta_{2i}\right)\left(\frac{1}{\zeta_{iE}} + \frac{1}{\zeta_{3E}\,x_E}\right),$$
$$i = 1, 2, \tag{7.83}$$

where δ_{ij} represents Kronecker's delta [24], i.e., $\delta_{ij} = 0$ or $\delta_{ij} = 1$ for $i \neq j$ or $i = j$, respectively.

Conditions $C_{2E} \neq 0$, $C_{4E} \neq 0$, $C_{1E} = C_{3E} = 0$. With regard to Equations (2.33), (2.34), (7.5)–(7.12), (2.22), (2.23), we get

$$\varepsilon_{nE} = -c_{3E}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) x_n^{c_{3E}-1}, \tag{7.84}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\left[\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) x_n^{c_{3E}-1} + \frac{p_{1n} - p_{2n}}{\zeta_{3E}\,x_n}\right], \tag{7.85}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\left[x_n^{c_{3E}-1}\,\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{1}{x_n}\,\frac{\partial}{\partial\varphi}\left(\frac{p_{1n} - p_{2n}}{\zeta_{3E}}\right)\right], \tag{7.86}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\Theta\left[x_n^{c_{3E}-1}\,\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{1}{x_n}\,\frac{\partial}{\partial\nu}\left(\frac{p_{1n} - p_{2n}}{\zeta_{3E}}\right)\right], \tag{7.87}$$

$$\sigma_{nE} = -\left[\left[\left(c_{1E} + c_{2E}\right)c_{3E} - 2\,c_{2E}\right]\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) x_n^{c_{3E}-1} - \frac{2\,c_{2E}\left(p_{1n} - p_{2n}\right)}{\zeta_{3E}\,x_n}\right], \tag{7.88}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\left[\left(c_{1E} - c_{2E}\,c_{3E}\right)\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) x_n^{c_{3E}-1} + \frac{c_{1E}\left(p_{1n} - p_{2n}\right)}{\zeta_{3E}\,x_n}\right], \tag{7.89}$$

$$w_E = \kappa_{2E}\,x_n^{2(c_{3E}-1)} + \frac{\kappa_{4E}}{x_n^2} + \kappa_{9E}\,x_n^{c_{3E}-1} \tag{7.90}$$

$$W_E = 4 \int_0^{\pi/2} \int_0^{\pi/2} \left[\frac{\kappa_{2E}}{2c_{3E}+1} \left(x_E^{2c_{3E}+1} - x_{IN}^{2c_{3E}+1} \right) + \kappa_{4E} \left(x_E - x_{IN} \right) \right.$$
$$\left. + \frac{\kappa_{9E}}{c_{3E}+2} \left(x_E^{c_{3E}+2} - x_{IN}^{c_{3E}+2} \right) \right] d\varphi \, d\nu, \tag{7.91}$$

where Θ, x_{IN}, x_E and s_{44E}, c_{iE} are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_{iE} $(i = 1,2,3)$, κ_{9E} (see Equation (7.13)) have the forms

$$\begin{aligned}
\zeta_{iE} &= \frac{1}{x_{IN}} \left[c_{3E} \left(c_{1E} + c_{2E} \right) - 2c_{2E} \right] \left(x_{IN}^{c_{3E}-1} - x_E^{c_{3E}-1} \right), \\
\zeta_{2E} &= \frac{1}{x_E} \left[c_{3E} \left(c_{1E} + c_{2E} \right) - 2c_{2E} \right] \left(x_{IN}^{c_{3E}-1} - x_E^{c_{3E}-1} \right), \\
\zeta_{3E} &= \frac{x_{IN}^{c_{3E}-1} - x_E^{c_{3E}-1}}{x_{IN} \, x_E}, \\
\kappa_{9E} &= \left(c_{1E} - c_{2E} \, c_{3E} \right) C_{2E} \, C_{4E} + \frac{1}{s_{44E}} \left(\frac{\partial C_{2E}}{\partial \varphi} \frac{\partial C_{4E}}{\partial \varphi} + \Theta^2 \frac{\partial C_{2E}}{\partial \nu} \frac{\partial C_{4E}}{\partial \nu} \right).
\end{aligned} \tag{7.92}$$

The coefficients κ_{2E}, κ_{2E} are given by Equations (7.82), (6.61), respectively, where ζ_{iE} $(i = 1,2,3)$ in Equations (7.82), (6.61) is given by Equation (7.92). The normal stresses p_{1n}, p_{2n} are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (7.23), (7.33), (7.43), (7.53), respectively. With regard to $\left(\varepsilon_{\varphi E} \right)_{x_n = x_{IN}} = -\left(p_{1n} \rho_{11E} + p_{2n} \rho_{12E} \right)$, $\left(\varepsilon_{\varphi E} \right)_{x_n = x_E} = -\left(p_{1n} \rho_{21E} + p_{2n} \rho_{22E} \right)$ and Equation (7.85), the coefficient ρ_{ijE} $(i, j = 1,2)$ in Equations (2.25), (2.26) is derived as

$$\rho_{1iE} = \left(\delta_{1i} - \delta_{2i} \right) \left(\frac{x_{IN}^{c_{3E}-1}}{\zeta_{iE}} + \frac{1}{\zeta_{3E} \, x_{IN}} \right), \quad \rho_{2iE} = \left(\delta_{1i} - \delta_{2i} \right) \left(\frac{x_E^{c_{3E}-1}}{\zeta_{iE}} + \frac{1}{\zeta_{3E} \, x_E} \right),$$
$$i = 1,2, \tag{7.93}$$

where δ_{ij} represents Kronecker's delta [24], i.e., $\delta_{ij} = 0$ or $\delta_{ij} = 1$ for $i \neq j$ or $i = j$, respectively.

Conditions $C_{3E} \neq 0$, $C_{4E} \neq 0$, $C_{1E} = C_{2E} = 0$. With regard to Equations (2.33), (2.34), (7.5)–(7.12), (2.22), (2.23), we get

$$\varepsilon_{nE} = 2 \left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}} \right) \frac{1}{x_n^3}, \tag{7.94}$$

$$\varepsilon_{\varphi E} = \varepsilon_{\theta E} = -\left[\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\frac{1}{x_n^3} + \left(\frac{p_{1n}}{\zeta_{3E}} - \frac{p_{2n}}{\zeta_{4E}}\right)\frac{1}{x_n}\right], \tag{7.95}$$

$$\varepsilon_{n\varphi E} = s_{44E}\,\sigma_{n\varphi E} = -\left[\frac{1}{x_n^3}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{1}{x_n}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{3E}} - \frac{p_{2n}}{\zeta_{4E}}\right)\right], \tag{7.96}$$

$$\varepsilon_{n\theta E} = s_{44E}\,\sigma_{n\theta E} = -\Theta\left[\frac{1}{x_n^3}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right) + \frac{1}{x_n}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{3E}} - \frac{p_{2n}}{\zeta_{4E}}\right)\right], \tag{7.97}$$

$$\sigma_{nE} = -2\left[(c_{1E} + 2\,c_{2E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\frac{1}{x_n^3} + c_{2E}\left(\frac{p_{1n}}{\zeta_{3E}} - \frac{p_{2n}}{\zeta_{4E}}\right)\frac{1}{x_n}\right], \tag{7.98}$$

$$\sigma_{\varphi E} = \sigma_{\theta E} = -\left[(c_{1E} + 2\,c_{2E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\frac{1}{x_n^3}c_{1E}\left(\frac{p_{1n}}{\zeta_{3E}} - \frac{p_{2n}}{\zeta_{4E}}\right) + \frac{1}{x_n}\right], \tag{7.99}$$

$$w_E = \frac{\kappa_{3E}}{x_n^6} + \frac{\kappa_{4E}}{x_n^2} + \frac{\kappa_{10E}}{x_n^4}, \tag{7.100}$$

$$W_E = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{3E}}{3}\left(\frac{1}{x_{IN}^3} - \frac{1}{x_E^3}\right) + \kappa_{4E}\,(x_E - x_{IN}) + \frac{\kappa_{10E}}{2}\left(\frac{1}{x_{IN}} - \frac{1}{x_E}\right)\right] d\varphi\, d\nu, \tag{7.101}$$

where Θ, x_{IN}, x_E and s_{44E}, c_{iE} $(i = 1,2,3)$ are given by Equations (1.21), (1.22) and (2.13), (2.18), respectively, and ζ_{iE} $(i = 1, \ldots, 4)$, κ_{jE} $(j = 1,4,10$; see Equations (6.12), (7.13)) have the forms

$$\zeta_{1E} = \frac{2\,(c_{1E} + 2c_{2E})}{x_{IN}^3}\left[\left(\frac{x_{IN}}{x_E}\right)^2 - 1\right], \quad \zeta_{2E} = \frac{2\,(c_{1E} + 2c_{2E})}{x_E^3}\left[1 - \left(\frac{x_E}{x_{IN}}\right)^2\right],$$

$$\zeta_{3E} = \frac{2c_{2E}}{x_{IN}}\left[\left(\frac{x_E}{x_{IN}}\right)^2 - 1\right], \quad \zeta_{4E} = \frac{2c_{2E}}{x_E}\left[\left(\frac{x_{IN}}{x_E}\right)^2 - 1\right].$$

$$\kappa_{3E} = 3\,(c_{1E} + 2\,c_{2E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)^2 + \frac{1}{s_{44E}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\right]^2\right\},$$

$$\kappa_{4E} = (c_{1E} - c_{2E})\,(2 + c_{3E})\left(\frac{p_{1n}}{\zeta_{1E}} - \frac{p_{2n}}{\zeta_{2E}}\right)\left(\frac{p_{1n}}{\zeta_{3E}} - \frac{p_{2n}}{\zeta_{4E}}\right)$$

$$
\begin{aligned}
&+\frac{2}{s_{44E}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right)\\
&+\frac{2\Theta^2}{s_{44E}}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right),\\
\kappa_{10E}={}&(c_{1E}+2c_{2E})\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right)\\
&+\frac{1}{s_{44E}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right)\\
&+\frac{\Theta^2}{s_{44E}}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{1E}}-\frac{p_{2n}}{\zeta_{2E}}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{1n}}{\zeta_{3E}}-\frac{p_{2n}}{\zeta_{4E}}\right),
\end{aligned}
\tag{7.102}
$$

The normal stresses p_{1n}, p_{2n} are given by Equations (2.25), (2.26), where ρ_{IN} and ρ_M in Equations (2.25), (2.26) are given by Equation (6.89) and (7.23), (7.33), (7.43), (7.53), respectively. With regard to $\left(\varepsilon_{\varphi E}\right)_{x_n=x_{IN}}=-(p_{1n}\rho_{11E}+p_{2n}\rho_{12E})$, $\left(\varepsilon_{\varphi E}\right)_{x_n=x_E}=-(p_{1n}\rho_{21E}+p_{2n}\rho_{22E})$ and Equation (7.95), the coefficient ρ_{ijE} ($i,j=$ 1,2) in Equations (2.25), (2.26) is derived as

$$
\rho_{1iE}=(\delta_{1i}-\delta_{2i})\left(\frac{1}{\zeta_{iE}R_1^3}+\frac{1}{\zeta_{3E}R_1}\right),\quad \rho_{2jE}=(\delta_{1i}-\delta_{2i})\left(\frac{1}{\zeta_{iE}R_2^3}+\frac{1}{\zeta_{3E}R_2}\right),
$$
$$
i=1,2, \tag{7.103}
$$

where δ_{ij} represents Kronecker's delta [24], i.e., $\delta_{ij}=0$ or $\delta_{ij}=1$ for $i\neq j$ or $i=j$, respectively.

7.3 Condition $\beta_{IN}\neq\beta_E=\beta_M$

If $\beta_{IN}\neq\beta_E=\beta_M$, then $|u_{nE}|=f(x_n)$ represents a decreasing function of $x_n\in\langle x_{IN},x_E\rangle$, and then $C_{1E}=C_{4E}=0$, $C_{2E}\neq 0$, $C_{3E}\neq 0$ (see Equation (7.5)), respectively. With regard to one boundary condition (see Equation (2.33)), the thermal stresses in the ellipsoidal inclusion are determined on the conditions $C_{2E}\neq 0$, $C_{3E}=0$; $C_{23}\neq 0$, $C_{2E}=0$. With regard to Equations (6.4), (7.5), the thermal strains and stresses in the ellipsoidal envelope are determined in Sections 4.3.2 and 6.3.2 for $C_{2E}\neq 0$, $C_{3E}=0$ and $C_{23}\neq 0$, $C_{2E}=0$, respectively. The normal stress p_{1n} in Section 4.3.2 is given by Equation (2.27), where ρ_{IN} and ρ_{1ME} in Equation (2.27) are given by Equations (6.89) and (4.130), respectively.

7.3.1 Matrix

The integration constants $C_{1M}, C_{2M}, C_{3M}, C_{4M}$ for the matrix (see Equation (7.5)) are determined by the boundary conditions (2.30), (2.31) or (2.30)–(2.32). The boundary conditions result in the following combinations of C_{1M}, C_{2M}, C_{3M}, C_{4M}, where the combinations of C_{1M}, C_{2M}, C_{3M} are presented in Section 6.3.1. Finally, such a combination is considered to exhibit a minimum value of the elastic energy W_C of the cubic cell (see Equation (2.23)).

Conditions $C_{1M} \neq 0$, $C_{4M} \neq 0$, $C_{2M} = C_{3M} = 0$. With regard to Equations (2.30), (2.31), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}, \tag{7.104}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}\left(1 - \frac{x_M}{x_n}\right), \tag{7.105}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right) - \frac{1}{x_n}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}\,x_M}{\zeta_M}\right)\right], \tag{7.106}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right) - \frac{1}{x_n}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}\,x_M}{\zeta_M}\right)\right], \tag{7.107}$$

$$\sigma_{nM} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}\left(c_{1M} - c_{2M} + \frac{2\,c_{2M}\,x_M}{x_n}\right), \tag{7.108}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -\frac{p_{1n}\rho_{2ME}}{\zeta_M}\left(c_{1M} - c_{2M} - \frac{c_{1M}\,x_M}{x_n}\right), \tag{7.109}$$

$$w_M = \kappa_{1M} + \frac{\kappa_{4M}}{x_n^2} + \frac{\kappa_{8M}}{x_n}, \tag{7.110}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \kappa_{4M}\left(x_M - x_E\right) + \frac{\kappa_{8M}}{2}\left(x_M^2 - x_E^2\right)\right] d\varphi\, d\nu, \tag{7.111}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i = 1,2)$ are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_M, κ_{jM} $(j = 4,8$; see Equation (7.13)) have the forms

$$\zeta_M = 1 - \frac{x_M}{x_E},$$

$$\kappa_{4M} = c_{1M}\left(\frac{p_{1n}\rho_{2ME}x_M}{\zeta_M}\right)^2 + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}x_M}{\zeta_M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}x_M}{\zeta_M}\right)\right]^2\right\},$$

$$\kappa_{8M} = x_M\left(c_{2M}-c_{1M}\right)\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)^2 - \frac{1}{s_{44M}}\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)\frac{\partial}{\partial\varphi}\left(\frac{p_{1n}\rho_{2ME}x_M}{\zeta_M}\right) - \frac{\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}}{\zeta_M}\right)\frac{\partial}{\partial\nu}\left(\frac{p_{1n}\rho_{2ME}x_M}{\zeta_M}\right). \tag{7.112}$$

The coefficient κ_{1M} is given by Equation (6.98), where ζ_M in Equation (6.98) is given by Equation (7.112). The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{2M} \neq 0$, $C_{4M} \neq 0$, $C_{1M} = C_{3M} = 0$. With regard to Equations (2.30), (2.31), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -p_{1n}\rho_{2ME}\zeta_{1M}c_{3M}x_n^{c_{3M}-1}, \tag{7.113}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -p_{1n}\rho_{2ME}\left(\zeta_{1M}x_n^{c_{3M}-1} + \frac{\zeta_{2M}}{x_n}\right), \tag{7.114}$$

$$\varepsilon_{n\varphi M} = s_{44M}\sigma_{n\varphi M} = -\left[x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{1M}\right) + \frac{1}{x_n}\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{2M}\right)\right], \tag{7.115}$$

$$\varepsilon_{n\theta M} = s_{44M}\sigma_{n\theta M} = -\Theta\left[x_n^{c_{3M}-1}\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{1M}\right) + \frac{1}{x_n}\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{2M}\right)\right], \tag{7.116}$$

$$\sigma_{nM} = -p_{1n}\rho_{2ME}\left\{\zeta_{1M}\left[c_{3M}\left(c_{1M}+c_{2M}\right)-2c_{2M}\right]x_n^{c_{3M}-1} - \frac{2\zeta_{2M}c_{2M}}{x_n}\right\}, \tag{7.117}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -p_{1n}\rho_{2ME}\left[\zeta_{1M}\left(c_{1M}-c_{2M}c_{3M}\right)x_n^{c_{3M}-1} + \frac{\zeta_{2M}c_{1M}}{x_n}\right], \tag{7.118}$$

$$w_M = \kappa_{2M} x_n^{2(c_{3M}-1)} + \frac{\kappa_{4M}}{x_n^2} + \kappa_{9M} x_n^{c_{3M}-1}, \tag{7.119}$$

$$W_M = 4 \int_0^{\pi/2}\int_0^{\pi/2} \left[\frac{\kappa_{2M}}{2c_{3M}+1} \left(x_M^{2c_{3M}+1} - x_E^{2c_{3M}+1} \right) + \kappa_{4M} \left(x_M - x_E \right) \right.$$
$$\left. + \frac{\kappa_{9M}}{c_{3M}+2} \left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2} \right) \right] d\varphi\, d\nu, \tag{7.120}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} $(i=1,2,3)$ are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_{jM} $(j=1,2)$, κ_{kM} $(k=2,4$; see Equation (6.13)), κ_{9M} (see Equation (7.13)) have the forms

$$\zeta_{1M} = \frac{x_E}{x_E^{c_{3M}} - x_M^{c_{3M}}}, \quad \zeta_{2M} = -\frac{x_E x_M^{c_{3M}}}{x_E^{c_{3M}} - x_M^{c_{3M}}},$$
$$\kappa_{2M} = \left[\frac{(c_{1M}+c_{2M})c_{3M}^2}{2} + c_{1M} - 2c_{2M}c_{3M} \right] (p_{1n}\rho_{2ME}\zeta_{1M})^2$$
$$+ \frac{1}{s_{44M}} \left\{ \left[\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{1M}) \right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{1M}) \right]^2 \right\},$$
$$\kappa_{4M} = c_{1M}\,(p_{1n}\rho_{2ME}\zeta_{2M})^2$$
$$+ \frac{1}{s_{44M}} \left\{ \left[\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{2M}) \right]^2 + \Theta^2 \left[\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{2M}) \right]^2 \right\},$$
$$\kappa_{9M} = \zeta_{1M}\zeta_{2M}(c_{1M} - c_{2M}c_{3M})(p_{1n}\rho_{2ME})^2$$
$$+ \frac{1}{s_{44M}} \frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{1M}) \frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{2M})$$
$$+ \frac{\Theta^2}{s_{44M}} \frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{1M}) \frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{2M}). \tag{7.121}$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{3M} \neq 0$, $C_{4M} \neq 0$, $C_{1M} = C_{2M} = 0$. With regard to Equations (2.30), (2.31), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = \frac{2p_{1n}\rho_{2ME}\zeta_{1M}}{x_n^3}, \tag{7.122}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -p_{1n}\,\rho_{2ME}\left(\frac{\zeta_{1M}}{x_n^3} + \frac{\zeta_{2M}}{x_n}\right), \tag{7.123}$$

$$\varepsilon_{n\varphi M} = s_{44M}\,\sigma_{n\varphi M} = -\left[\frac{1}{x_n^3}\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right) + \frac{1}{x_n}\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right)\right], \tag{7.124}$$

$$\varepsilon_{n\theta M} = s_{44M}\,\sigma_{n\theta M} = -\Theta\left[\frac{1}{x_n^3}\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right) + \frac{1}{x_n}\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right)\right], \tag{7.125}$$

$$\sigma_{nM} = 2p_{1n}\,\rho_{2ME}\left[\frac{\zeta_{1M}\left(c_{1M} + 2c_{2M}\right)}{x_n^3} + \frac{\zeta_{2M}\,c_{2M}}{x_n}\right], \tag{7.126}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -p_{1n}\,\rho_{2ME}\left[\frac{\zeta_{1M}\left(c_{1M} + 2c_{2M}\right)}{x_n^3} + \frac{\zeta_{2M}\,c_{1M}}{x_n}\right], \tag{7.127}$$

$$w_M = \frac{\kappa_{3M}}{x_n^6} + \frac{\kappa_{4M}}{x_n^2} + \frac{\kappa_{10M}}{x_n^4}, \tag{7.128}$$

$$W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\left[\frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) + \kappa_{4M}\left(x_M - x_E\right) + \kappa_{10M}\left(\frac{1}{x_E} - \frac{1}{x_M}\right)\right] d\varphi\, d\nu, \tag{7.129}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} ($i = 1,2,3$) are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_{jM} ($j = 1,2$), κ_{3M} (see Equation (6.13)), κ_{10M} (see Equation (7.13)) have the forms

$$\begin{aligned}
\zeta_{1M} &= \frac{x_E^3}{1 - \left(\frac{x_E}{x_M}\right)^2}, \quad \zeta_{2M} = \frac{x_E}{1 - \left(\frac{x_M}{x_E}\right)^2}, \\
\kappa_{3M} &= 3\left(c_{1M} + 2c_{2M}\right)\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right)^2 \\
&\quad + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right)\right]^2\right\}, \\
\kappa_{10M} &= \zeta_{1M}\,\zeta_{2M}\left(c_{1M} + 2c_{2M}\right)\left(p_{1n}\,\rho_{2ME}\right)^2 \\
&\quad + \frac{1}{s_{44M}}\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right)\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right) \\
&\quad + \frac{\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right)\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right).
\end{aligned} \tag{7.130}$$

The coefficient κ_{4M} is given by Equation (7.121), where ζ_M in Equation (7.121) is given by Equation (7.130). The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{1M} \neq 0, C_{2M} \neq 0, C_{4M} \neq 0, C_{3M} = 0$. With regard to Equations (2.30)–(2.32), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -p_{1n}\rho_{2ME}\left(\zeta_{1M} + \zeta_{2M} c_{3M} x_n^{c_{3M}-1}\right), \tag{7.131}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -p_{1n}\rho_{2ME}\left(\zeta_{1M} + \zeta_{2M} x_n^{c_{3M}-1} + \frac{\zeta_{3M}}{x_n}\right), \tag{7.132}$$

$$\begin{aligned}\varepsilon_{n\varphi M} = s_{44M}\sigma_{n\varphi M} = -\bigg[&\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{1M}\right) + x_n^{c_{3M}-1}\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{2M}\right)\\ &+ \frac{1}{x_n}\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{3M}\right)\bigg],\end{aligned} \tag{7.133}$$

$$\begin{aligned}\varepsilon_{n\theta M} = s_{44M}\sigma_{n\theta M} = -\Theta\bigg[&\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{1M}\right) + x_n^{c_{3M}-1}\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{2M}\right)\\ &+ \frac{1}{x_n}\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{3M}\right)\bigg],\end{aligned} \tag{7.134}$$

$$\begin{aligned}\sigma_{nM} = -p_{1n}\rho_{2ME}\bigg\{&\zeta_{1M}\left(c_{1M} - c_{2M}\right) + \zeta_{2M}\left[c_{3M}\left(c_{1M} + c_{2M}\right) - 2c_{2M}\right] x_n^{c_{3M}-1}\\ &- \frac{2\zeta_{3M} c_{2M}}{x_n}\bigg\},\end{aligned} \tag{7.135}$$

$$\begin{aligned}\sigma_{\varphi M} = \sigma_{\theta M} = -p_{1n}\rho_{2ME}\bigg[&\zeta_{1M}\left(c_{1M} - c_{2M}\right) + \zeta_{2M}\left(c_{1M} - c_{2M}c_{3M}\right) x_n^{c_{3M}-1}\\ &+ \frac{\zeta_{3M} c_{1M}}{x_n}\bigg],\end{aligned} \tag{7.136}$$

$$w_M = \kappa_{1M} + \kappa_{2M} x_n^{2(c_{3M}-1)} + \frac{\kappa_{4M}}{x_n^2} + \left(\kappa_{5M} + \kappa_{9M}\right) x_n^{c_{3M}-1} + \frac{\kappa_{8M}}{x_n}, \tag{7.137}$$

$$\begin{aligned}W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\bigg[&\frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \frac{\kappa_{2M}}{2c_{3M}+1}\left(x_M^{2c_{3M}+1} - x_E^{2c_{3M}+1}\right) + \kappa_{4M}\left(x_M - x_E\right)\\ &+ \frac{\kappa_{5M} + \kappa_{9M}}{c_{3M}+2}\left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2}\right) + \frac{\kappa_{8M}}{2}\left(x_M^2 - x_E^2\right)\bigg]\, d\varphi\, d\nu,\end{aligned} \tag{7.138}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} ($i=1,2,3$) are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_{jM} ($j=1,2,3$), κ_{1M}, κ_{2M} (see Equation (6.13)), κ_{kM} ($k=4,5,8,9$; see Equation (7.13)) have the forms

$$
\begin{aligned}
\zeta_{1M} &= -\frac{c_{3M}x_M^{c_{3M}-1}}{\zeta_M}, \quad \zeta_{2M} = \frac{1}{\zeta_M}, \quad \zeta_{3M} = \frac{(c_{3M}-1)\,x_M^{c_{3M}}}{\zeta_M},\\
\zeta_M &= \frac{(c_{3M}-1)\,x_M^{c_{3M}}}{x_E} - c_{3M}x_M^{c_{3M}-1} + x_E^{c_{3E}-1},\\
\kappa_{1M} &= \frac{3\,(c_{1M}-c_{2M})\,(p_{1n}\rho_{2ME}\zeta_{1M})^2}{2}\\
&\quad + \frac{1}{s_{44}}\left\{\left[\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{1M})\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{1M})\right]^2\right\},\\
\kappa_{2M} &= \left[\frac{(c_{1M}+c_{2M})\,c_{3M}^2}{2} + c_{1M} - 2\,c_{2M}\,c_{3M}\right](p_{1n}\rho_{2ME}\zeta_{2M})^2\\
&\quad + \frac{1}{s_{44}}\left\{\left[\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{2M})\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{2M})\right]^2\right\},\\
\kappa_{4M} &= c_{1M}\,(p_{1n}\rho_{2ME}\zeta_{3M})^2\\
&\quad + \frac{1}{s_{44}}\left\{\left[\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{3M})\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{3M})\right]^2\right\},\\
\kappa_{5M} &= \zeta_{1M}\zeta_{2M}\,(c_{1M}-c_{2M})\,(2+c_{3M})\,C_{1M}C_{2M}\,(p_{1n}\rho_{2ME})^2\\
&\quad + \frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{1M})\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{2M})\\
&\quad + \frac{2\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{1M})\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{2M})\\
\kappa_{8M} &= \zeta_{1M}\zeta_{3M}\,(c_{1M}-c_{2M})\,(p_{1n}\rho_{2ME})^2\\
&\quad + \frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{1M})\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{3M})\\
&\quad + \frac{2\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{1M})\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{3M})\\
\kappa_{9M} &= \zeta_{2M}\zeta_{3M}\,(c_{1M}-c_{2M}\,c_{3M})\,(p_{1n}\rho_{2ME})^2\\
&\quad + \frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{2M})\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{3M})\\
&\quad + \frac{2\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{2M})\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{3M}).
\end{aligned}
\tag{7.139}
$$

The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{1M} \neq 0, C_{3M} \neq 0, C_{4M} \neq 0, C_{2M} = 0$. With regard to Equations (2.30)–(2.32), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM} = -p_{1n}\rho_{2ME}\left(\zeta_{1M} - \frac{2\zeta_{2M}}{x_n^3}\right), \tag{7.140}$$

$$\varepsilon_{\varphi M} = \varepsilon_{\theta M} = -p_{1n}\rho_{2ME}\left(\zeta_{1M} + \frac{\zeta_{2M}}{x_n^3} + \frac{\zeta_{3M}}{x_n}\right), \tag{7.141}$$

$$\begin{aligned}\varepsilon_{n\varphi M} = s_{44M}\sigma_{n\varphi M} = -\Bigg[&\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{1M}) + \frac{1}{x_n^3}\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{2M})\\ &+ \frac{1}{x_n}\frac{\partial}{\partial\varphi}(p_{1n}\rho_{2ME}\zeta_{3M})\Bigg],\end{aligned} \tag{7.142}$$

$$\begin{aligned}\varepsilon_{n\theta M} = s_{44M}\sigma_{n\theta M} = -\Theta\Bigg[&\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{1M}) + \frac{1}{x_n^3}\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{2M})\\ &+ \frac{1}{x_n}\frac{\partial}{\partial\nu}(p_{1n}\rho_{2ME}\zeta_{3M})\Bigg],\end{aligned} \tag{7.143}$$

$$\sigma_{nM} = -p_{1n}\rho_{2ME}\left[\zeta_{1M}(c_{1M} - c_{2M}) - \frac{2\zeta_{2M}(c_{1M} + 2c_{2M})}{x_n^3} - \frac{2\zeta_{3M}c_{2M}}{x_n}\right], \tag{7.144}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -p_{1n}\rho_{2ME}\left[\zeta_{1M}(c_{1M} - c_{2M}) + \frac{\zeta_{2M}(c_{1M} + 2c_{2M})}{x_n^3} + \frac{\zeta_{3M}c_{1M}}{x_n}\right], \tag{7.145}$$

$$w_M = \kappa_{1M} + \frac{\kappa_{3M}}{x_n^6} + \frac{\kappa_{4M}}{x_n^2} + \frac{\kappa_{6M}}{x_n^3} + \frac{\kappa_{8M}}{x_n} + \frac{\kappa_{10M}}{x_n^4}, \tag{7.146}$$

$$\begin{aligned}W_M = 4\int_0^{\pi/2}\int_0^{\pi/2}\Bigg[&\frac{\kappa_{1M}}{3}\left(x_M^3 - x_E^3\right) + \frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) + \kappa_{4M}(x_M - x_E)\\ &+ \kappa_{6M}\ln\left(\frac{x_M}{x_E}\right) + \frac{\kappa_{8M}}{2}\left(x_M^2 - x_E^2\right) + \kappa_{10M}\left(\frac{1}{x_E} - \frac{1}{x_M}\right)\Bigg]\, d\varphi\, d\nu,\end{aligned} \tag{7.147}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} ($i=1,2,3$) are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_{jM} ($j=1,2,3$), κ_{3M} (see Equation (6.13)), κ_{kM} ($k=6,10$; see Equation (7.13)) have the forms

$$\begin{aligned}
&\zeta_{1M}=\frac{2}{\zeta_M x_M^3},\quad \zeta_{2M}=\frac{1}{\zeta_M},\quad \zeta_{3M}=-\frac{3}{\zeta_M x_M^2},\quad \zeta_M=\frac{2}{x_M^3}+\frac{1}{x_E^3}-\frac{3}{x_E x_M^2},\\
&\kappa_{3M}=3\left(c_{1M}+2c_{2M}\right)\left(p_{1n}\rho_{2ME}\zeta_{2M}\right)^2\\
&\qquad+\frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{2M}\right)\right]^2+\Theta^2\left[\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{2M}\right)\right]^2\right\},\\
&\kappa_{6M}=\frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{1M}\right)\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{3M}\right)\\
&\qquad+\frac{2\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{1M}\right)\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{3M}\right),\\
&\kappa_{10M}=\zeta_{2M}\zeta_{3M}\left(c_{1M}+2c_{2M}\right)\left(p_{1n}\rho_{2ME}\right)^2\\
&\qquad+\frac{1}{s_{44M}}\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{2M}\right)\frac{\partial}{\partial\varphi}\left(p_{1n}\rho_{2ME}\zeta_{3M}\right)\\
&\qquad+\frac{\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{2M}\right)\frac{\partial}{\partial\nu}\left(p_{1n}\rho_{2ME}\zeta_{3M}\right).
\end{aligned}\tag{7.148}$$

The coefficients κ_{1M}, κ_{4M}, κ_{8M} are given by Equation (7.121), where ζ_{iM} ($i=1,2,3$) in Equation (7.121) is given by Equation (7.148). The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

Conditions $C_{2M}\neq 0, C_{3M}\neq 0, C_{4M}\neq 0, C_{1M}=0$. With regard to Equations (2.30)–(2.32), (7.5)–(7.12), (2.21), (2.22), (2.23), we get

$$\varepsilon_{nM}=-p_{1n}\rho_{2ME}\left(\zeta_{1M}c_{3M}x_n^{c_{3M}-1}-\frac{2\zeta_{2M}}{x_n^3}\right),\tag{7.149}$$

$$\varepsilon_{\varphi M}=\varepsilon_{\theta M}=-p_{1n}\rho_{2ME}\left(\zeta_{1M}x_n^{c_{3M}-1}+\frac{\zeta_{2M}}{x_n^3}+\frac{\zeta_{3M}}{x_n}\right),\tag{7.150}$$

$$\varepsilon_{n\varphi M}=s_{44M}\sigma_{n\varphi M}=-p_{1n}\rho_{2ME}\left(x_n^{c_{3M}-1}\frac{\partial\zeta_{1M}}{\partial\varphi}+\frac{1}{x_n^3}\frac{\partial\zeta_{2M}}{\partial\varphi}+\frac{1}{x_n}\frac{\partial\zeta_{3M}}{\partial\varphi}\right),\tag{7.151}$$

$$\varepsilon_{n\theta M}=s_{44M}\sigma_{n\theta M}=-\Theta p_{1n}\rho_{2ME}\left(x_n^{c_{3M}-1}\frac{\partial\zeta_{1M}}{\partial\nu}+\frac{1}{x_n^3}\frac{\partial\zeta_{2M}}{\partial\nu}+\frac{1}{x_n}\frac{\partial\zeta_{3M}}{\partial\nu}\right),\tag{7.152}$$

$$\sigma_{nM} = -p_{1n}\,\rho_{2ME}\left\{\zeta_{1M}\left[c_{3M}\left(c_{1M}+c_{2M}\right)-2c_{2M}\right]x_n^{c_{3M}-1} - \frac{2\zeta_{2M}\left(c_{1M}+2c_{2M}\right)}{x_n^3} - \frac{2\zeta_{3M}c_{2M}}{x_n}\right\}, \tag{7.153}$$

$$\sigma_{\varphi M} = \sigma_{\theta M} = -p_{1n}\,\rho_{2ME}\left[\zeta_{1M}\left(c_{1M}-c_{2M}c_{3M}\right)x_n^{c_{3M}-1} + \frac{\zeta_{2M}\left(c_{1M}+2c_{2M}\right)}{x_n^3} + \frac{\zeta_{3M}\,c_{1M}}{x_n}\right], \tag{7.154}$$

$$w_M = \kappa_{2M}\,x_n^{2(c_{3M}-1)} + \frac{\kappa_{3M}}{x_n^6} + \frac{\kappa_{4M}}{x_n^2} + \kappa_{7M}\,x_n^{c_{3M}-4} + \kappa_{9M}\,x_n^{c_{3M}-1} + \frac{\kappa_{10M}}{x_n^4}, \tag{7.155}$$

$$\begin{aligned} W_M = 4\int_0^{\pi/2}\int_0^{\pi/2} &\left[\frac{\kappa_{2M}}{2c_{3M}+1}\left(x_M^{2c_{3M}+1} - x_E^{2c_{3M}+1}\right) + \frac{\kappa_{3M}}{3}\left(\frac{1}{x_E^3} - \frac{1}{x_M^3}\right) + \kappa_{4M}\left(x_M - x_E\right)\right. \\ &+ \frac{\kappa_{7M}}{c_{3M}-1}\left(x_M^{c_{3M}-1} - x_E^{c_{3M}-1}\right) + \frac{\kappa_{9M}}{c_{3M}+2}\left(x_M^{c_{3M}+2} - x_E^{c_{3M}+2}\right) \\ &\left. + \kappa_{10M}\left(\frac{1}{x_E} - \frac{1}{x_M}\right)\right] d\varphi\, d\nu, \end{aligned} \tag{7.156}$$

where Θ, x_E, x_M and s_{44M}, c_{iM} ($i=1,2,3$) are given by Equations (1.21)–(1.17) and (2.13), (2.18), respectively, and ζ_{jM} ($j=1,2,3$), κ_{3M} (see Equation (6.13)), κ_{kM} ($k=3,7,9,10$; see Equation (7.13)) have the forms

$$\begin{aligned} &\zeta_{1M} = \frac{2}{\zeta_M x_M^3}, \quad \zeta_{2M} = \frac{c_{3M}\,x_m^{c_{3M}-1}}{\zeta_M}, \quad \zeta_{3M} = -\frac{\left(c_{3M}+2\right)x_M^{c_{3M}-3}}{\zeta_M}, \\ &\zeta_M = \frac{2x_E^{c_{3M}-1}}{x_M^3} + \frac{c_{3M}\,x_M^{c_{3M}-1}}{x_E^3} - \frac{\left(c_{3M}+2\right)x_M^{c_{3M}-3}}{x_E}, \\ &\kappa_{3M} = 3\left(c_{1M}+2c_{2M}\right)\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right)^2 \\ &\qquad + \frac{1}{s_{44M}}\left\{\left[\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right)\right]^2 + \Theta^2\left[\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right)\right]^2\right\}, \\ &\kappa_{7M} = \zeta_{1M}\,\zeta_{2M}\left[2c_{2M}\left(1-c_{3M}\right)-c_{1M}\right]\left(p_{1n}\,\rho_{2ME}\right)^2 \\ &\qquad + \frac{2}{s_{44M}}\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right)\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right) \end{aligned}$$

$$
\begin{aligned}
&+\frac{2\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right)\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right),\\
\kappa_{9M} &= \zeta_{1M}\,\zeta_{3M}\left(c_{1M}-c_{2M}\,c_{3M}\right)\left(p_{1n}\,\rho_{2ME}\right)^2\\
&+\frac{1}{s_{44M}}\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right)\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{3M}\right)\\
&+\frac{\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{1M}\right)\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{3M}\right),\\
\kappa_{10M} &= \zeta_{2M}\,\zeta_{3M}\left(c_{1M}+2\,c_{2M}\right)\left(p_{1n}\,\rho_{2ME}\right)^2\\
&+\frac{1}{s_{44M}}\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right)\frac{\partial}{\partial\varphi}\left(p_{1n}\,\rho_{2ME}\,\zeta_{3M}\right)\\
&+\frac{\Theta^2}{s_{44M}}\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{2M}\right)\frac{\partial}{\partial\nu}\left(p_{1n}\,\rho_{2ME}\,\zeta_{3M}\right).
\end{aligned}
\tag{7.157}
$$

The coefficients κ_{2M}, κ_{4M} are given by Equations (7.121), (7.139), where ζ_{iM} ($i=1,2,3$) in Equations (7.121), (7.139) is given by Equation (7.157). The normal stress p_{1n} is given by Equation (2.27), where ρ_{IN} and ρ_{2ME} in Equation (2.27) are given by Equations (6.89) and (3.53), (4.130), (6.135), respectively, with respect to minimum W_c (see Equation (2.23)).

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